Homework 7. Due Wednesday, April 16, 2008

Exercise 7.1.

- Solve Problem **4.4.7** from the book.
- Show that u is a periodic function in time, and find its period, i.e. find a (minimal) number $\tau > 0$ such that $u(x, t + \tau) = u(x, t)$ for all $x \in [0;\ L]$ and all $t > 0$.

Exercise 7.2. The fundamental frequency of a uniform string is given by the formula

$$\omega_1 \ = \ \frac{\pi}{L} \cdot \sqrt{\frac{T_0}{\rho_0}}.$$

Q1: What is the meaning of the parameters L, T_0, and ρ_0?

Q2: A guitar player has to tune the strings before the performance. Which parameters stay the same and which parameters change during this "tune-up"?

Q3: A right-handed guitar player makes his performance. He uses both hands to play guitar. What does he do with his right hand? What does he do with his left hand? Which parameter (the only one) changes due to his left-hand actions (consider only one string)? How does the fundamental frequency change due to these actions?

Exercise 7.3. Solve the problem **4.4.1**, parts **a)** and **b)**.

Exercise 7.4. Let $L = \pi$. Consider the eigenvalue problem

$$\begin{aligned} &\frac{d^2\phi}{dx^2} \ + \ \lambda \cdot \phi(x) \ = \ 0, \quad 0 < x < \pi, \\ &\frac{d\phi}{dx}(0) \ = \ 0, \quad \phi(\pi) \ = \ 0 \ . \end{aligned}$$

Q1. Show that this is a Sturm-Liouville problem. Namely, indicate what are $p = p(x)$ and $\sigma = \sigma(x)$.

Q2. Write down all eigenpairs (λ_n, ϕ_n). *Hint.* You have two choices. Choice 1: derive them by considering three cases: $\lambda < 0$, $\lambda = 0$, and $\lambda > 0$. Choice 2: this work has been already done in Exercise **1.6**(Homework 1, Exercise 6). Simply refer to this exercise and write down the formulas.

Q3. Show that the function $\phi_n(x)$ has exactly $n - 1$ zeroes on interval $(0;\ \pi)$. In order to confirm your result, sketch the graphs of the first four eigenfunctions.

Q4. On the same picture, sketch the graphs of the eigenfunctions ϕ_5 and ϕ_6. Mark the zeroes of ϕ_5 as crosses, and the zeroes of ϕ_6 as circles. What can you say about the location of these zeroes with respect to each other?

Q5. Consider the eigenvalue problem

$$\begin{aligned} &\frac{d}{dx}\left[p(x)\frac{d\phi}{dx}\right] \ + \ \lambda \cdot \sigma(x) \cdot \phi(x) \ = \ 0, \quad 0 < x < \pi, \\ &\frac{d\phi}{dx}(0) \ = \ 0, \quad \phi(\pi) \ = \ 0 \ . \end{aligned}$$

Here, p and σ are arbitrary positive functions. The BC are the same as in the questions **Q1-Q4**. Use your imagination and sketch the first four eigenfunctions of this problem (on the same picture).