Quiz 2: HW due 1/24/2008

2.5 pts each problem

1.1 (10) $y' = 2x + \ln x$

Set of all solutions:

$$y = \int 2x + \ln x \, dx = x^2 + x\ln x - \int x \frac{1}{x} dx$$

int. by parts

$$= x^2 + x\ln x - x + C$$

1.1 (16) $y'' + 2y' - 8y = 0 \qquad y = e^{rx}$

Find r: $y = e^{rx}$

$y' = re^{rx}$

$y'' = r^2 e^{rx}$

$$y'' + 2y' - 8y = 0$$

$$(r^2 + 2r - 8)e^{rx} = 0$$

$$r^2 + 2r - 8 = 0$$

$$(r+4)(r-2) = 0 \qquad r = 2, -4$$

$y = e^{2x}, e^{-4x}$

1.3 (6) a. $y = c_1 x^2 + c_2 x^2 \ln x$

$$x^2 y'' - 3xy' + 4y = 0$$

Plug in: $y' = 2c_1 x + c_2(2x\ln x + x)$

$y'' = 2c_1 + c_2(2\ln x + 3)$

$$x^2 y'' - 3xy' + 4y = x^2(2c_1 + c_2(2\ln x + 3)) - 3x(2c_1 x + c_2(2x\ln x + x)) + 4c_1 x^2 + 4c_2 x^2 \ln x = 0 \Rightarrow \text{gen. sol.}$$

$y = c_1 x^2 + c_2 x^2 \ln x$ $\qquad 0 = c_1 \cdot 0 + c_2 \cdot 0 = 0$

$y' = 2c_1 x + c_2 (2x \ln x + x)$ $\qquad 0 = 2c_1 \, 0 + c_2 \, 0 \quad 0$

Yes, <u>all</u> solutions satisfy these conditions

(b) $y(1) = 0 \quad y'(1) = 1$

$$0 = c_1 + c_2 \cdot 0 \qquad c_1 = 0 \qquad y = x^2 \ln x$$
$$1 = 2c_1 + c_2 (0 + 1) \qquad c_2 = 1$$

(d) $y(0) = 0 \quad y'(0) = 1$

$$0 = c_1 \, 0 + c_2 \cdot 0 = 0$$
$$1 \neq 2c_1 \cdot 0 + c_2 \cdot 0 = 0 \quad \Rightarrow \underline{\text{no solution}}$$

1.3

(16)

$$y = c_1 e^x + c_2 e^{-2x} \qquad (1)$$
$$y' = c_1 e^x - 2c_2 e^{-2x} \qquad (2)$$
$$y'' = c_1 e^x + 4c_2 e^{-2x} \qquad (3)$$

Eliminate c_1, c_2, c_3

$$\left.\begin{array}{ll} (1)-(2) & y - y' = 3c_2 e^{-2x} \\ (1)-(3) & y - y'' = -3c_2 e^{-2x} \end{array}\right\} \quad c_2 e^{-2x} = \frac{y - y'}{3} = \frac{y'' - y}{3}$$

$$\Rightarrow \quad y - y' = y'' - y \qquad \boxed{y'' + y - 2y = 0}$$