HW due 2/28/08

3.4

(4) $y'' - \frac{1+x}{x} y' + \frac{1}{x} y = x e^{2x}$

Check $\{ y_1 = 1+x \, , \, y_2 = e^x \}$ is a fundamental set:

$$(1+x)'' - \frac{1+x}{x}(1+x)' + \frac{1}{x}(1+x) = 0 - \frac{1+x}{x} + \frac{1+x}{x} = 0$$

$\Rightarrow$ y_1 satisfies $y'' - \frac{1+x}{x} y' + \frac{1}{x} y = 0$

$$(e^x)'' - \frac{1+x}{x}(e^x)' + \frac{1}{x} e^x = e^x \left[1 - \frac{1+x}{x} + \frac{1}{x} \right] = e^x \cdot 0 = 0$$

$\Rightarrow$ y_2 satisfies $y'' - \frac{1+x}{x} y' + \frac{1}{x} y = 0$

$\Rightarrow$ y_1, y_2 solutions of the reduced eq.

$$W = \begin{vmatrix} 1+x & e^x \\ 1 & e^x \end{vmatrix} = (1+x) e^x - e^x = x e^x \neq 0$$

$\Rightarrow$ $\{1+x, e^x\}$ fund. set

Find particular solution $z = u_1 (1+x) + u_2 e^x$

$$u_1' = - \frac{e^x \cdot x e^{2x}}{W} = - \frac{e^x x e^{2x}}{x e^x} = -e^{2x}$$

$$\Rightarrow \quad u_1 = -\frac{1}{2} e^{2x}$$

$$u_2' = \frac{(1+x)\, x e^{2x}}{W} = \frac{(1+x)\, x e^{2x}}{x e^x} = (1+x) e^x$$

$$u_2 = \int (1+x) e^x \, dx \underset{\text{int. by parts}}{=} (1+x) e^x - \int e^x \, dx = (1+x) e^x - e^x = x e^x$$

$$\Rightarrow \quad z = -\frac{1}{2} e^{2x} (1+x) + x e^x \cdot e^x = -\frac{1}{2} e^{2x} - \frac{1}{2} x e^{2x} + x e^{2x}$$
$$= \frac{1}{2} x e^{2x} - \frac{1}{2} e^{2x}$$

Gen. solution

$$y = C_1 (1+x) + C_2 e^x + \frac{1}{2} x e^{2x} - \frac{1}{2} e^{2x}$$

3.4 12) $y'' - 4y' + 4y = \frac{1}{3x} e^{2x}$

$(r-2)^2 \qquad y_1 = e^{2x} \qquad y_2 = xe^{2x}$

Use variation of parameters:

$z = ue^{2x} + vxe^{2x}$

$z' = \underline{u'e^{2x} + v'xe^{2x}} + 2ue^{2x} + ve^{2x} + 2vxe^{2x}$

Set $u'e^{2x} + v'xe^{2x} = 0$

$z'' = 2u'e^{2x} + 4ue^{2x} + v'e^{2x} + 2ve^{2x} + 4vxe^{2x} + 2v'xe^{2}$
$+ 2ve^{2x}$

$$z'' - 4z' + 4z = 2u'e^{2x} + v'e^{2x} + 2v'xe^{2x}$$
$$+ \cancel{4ue^{2x}} + \cancel{2ve^{2x}} + \cancel{4ue^{2x}} + \cancel{2ve^{2x}}$$
$$- 4(\cancel{2ue^{2x}} + \cancel{ve^{2x}} + \cancel{2vxe^{2x}})$$
$$+ 4(\cancel{ue^{2x}} + \cancel{vxe^{2x}})$$
$$= 2u'e^{2x} + v'(1+2x)e^{2x} = \frac{1}{3x}e^{2x}$$

Solve

$$\begin{cases} u'e^{2x} + v'xe^{2x} = 0 \\ 2u'e^{2x} + v'(1+2x)e^{2x} = \frac{1}{3x}e^{2x} \end{cases}$$

$$\left.\begin{cases} u' + xv' = 0 \\ 2u' + (1+2x)v' = \frac{1}{3x} \end{cases}\right\} \Rightarrow \begin{matrix} v' = \frac{1}{3x} & & v = \frac{1}{3}\ln x \\ & \Rightarrow & \\ u' = -\frac{1}{3} & & u = -\frac{1}{3}x \end{matrix}$$

Particular solution: $z = -\frac{1}{3}xe^{2x} + \frac{1}{3}x\ln x\, e^{2x}$

General solution

$$y = c_1e^{2x} + c_2xe^{2x} - \frac{1}{3}xe^{2x} + \frac{1}{3}x\ln x\, e^{2x}$$

3.5 5) $y'' + 2y' = 4\sin 2x$

$r^2 + 2r = 0$

$r(r+2) = 0 \qquad r = 0 \qquad r = -2$

$\{1, e^{-2x}\}$

Undetermined coefficients:

$z = A\sin 2x + B\cos 2x$

Plug in (*) to find A and B

$\Rightarrow A = B = -1/2$

then

$y = c_1 + c_2 e^{-2x} - \frac{1}{2}\cos 2x - \frac{1}{2}\sin 2x$

(3.5)

(26) $y'' + 5y' + 6y = 2e^{2x}\cos x - 3xe^{3x} + 5e^{-2x}$

$r^2 + 5r + 6 = 0$

$(r+3)(r+2) = 0$

$\{e^{-3x}, e^{-2x}\}$

Form of particular solution

$z = Ae^{2x}\cos x + Be^{2x}\sin x + (Cx+D)e^{3x} + Exe^{-2x}$