QUIZ 7

Math 3321

Name: Solutions **March 13, 2008**

Question 1. *(6pts)*
Find the solution of the initial value problem

$$\begin{cases} y'' + 3y' + 2y = 6e^x \\ y(0) = 2 \\ y'(0) = -1 \end{cases}$$

$$s^2Y - sy(0) - y'(0) + 3\cdot(sY - y(0)) + 2Y = \frac{6}{s-1}$$

$$(s^2+3s+2)Y - 2s + 1 - 6 = \frac{6}{s-1}$$

$$(s+2)(s+1)Y = \frac{6}{s-1} + 2s + 5$$

$$Y = \frac{6}{(s+2)(s+1)(s-1)} + \frac{2s+5}{(s+2)(s+1)}$$

$$\frac{6}{(s+2)(s+1)(s-1)} = \frac{A}{s+2} + \frac{B}{s+1} + \frac{C}{s-1} = \frac{2}{s+2} - \frac{3}{s+1} + \frac{1}{s-1}$$

$$6 = A(s+1)(s-1) + B(s+2)(s-1) + C(s+2)(s+1)$$

$s=1$ $\quad 6 = C\cdot 3\cdot 2 \quad C = 1$

$s=-1$ $\quad 6 = B\cdot 1\cdot(-2) \quad B = -3$

$s=-2$ $\quad 6 = A(-1)(-3) \quad A = 2$

$$\frac{2s+5}{(s+2)(s+1)} = \frac{A}{s+2} + \frac{B}{s+1} = -\frac{1}{s+2} + \frac{3}{s+1}$$

$$2s+5 = A(s+1) + B(s+2)$$

$s=-2$ $\quad -4+5 = -A \quad A = -1$

$s=-1$ $\quad -2+5 = B. \quad B = 3$

Hence, Y becomes

$$Y = \frac{2}{s+2} - \frac{3}{s+1} + \frac{1}{s-1} - \frac{1}{s+2} + \frac{3}{s+1}$$

$$= \frac{1}{s+2} + \frac{1}{s-1}$$

$$y = \mathcal{L}^{-1}\left[\frac{1}{s+2}\right] + \mathcal{L}^{-1}\left[\frac{1}{s-1}\right]$$

$$= e^{-2x} + e^{x}$$

Question 2. *(4pts) Express the function*

$$f(x) = \begin{cases} x^2 & 0 \le x < 3 \\ 3x & x \ge 3 \end{cases}$$

in terms of the unit step function and its translations and then, find its Laplace transform $\mathcal{L}[f]$.

$$f_1(x) = \begin{cases} x^2 & 0 \le x < 3 \\ 0 & x \ge 3 \end{cases} = x^2 - x^2 u(x-3)$$

$$f_2(x) = \begin{cases} 0 & 0 \le x < 3 \\ 3x & x \ge 3 \end{cases} = 3x\, u(x-3)$$

$$f(x) = x^2 - x^2 u(x-3) + 3x\, u(x-3)$$

$$= x^2 - (\underline{\underline{x-3}}+3)^2 u(x-3) + 3(\underline{\underline{x-3}}+3)\, u(x-3)$$

$$= x^2 - [\underline{\underline{(x-3)}}^2 + 6\underline{\underline{(x-3)}} + 9]\, u(x-3) + [3\underline{\underline{(x-3)}} + 9]\, u(x-3)$$

$$= x^2 + [3\underline{\underline{(x-3)}} + \cancel{9} - \underline{\underline{(x-3)}}^2 - 6\underline{\underline{(x-3)}} - \cancel{9}]\, u(x-3)$$

$$= x^2 + [-(x-3)^2 - 3(x-3)]\, u(x-3)$$

$$\begin{array}{cc} \downarrow & \\ x^2 & -3x \\ \downarrow & \downarrow \\ \frac{2}{s^3} & -\frac{3}{s^2} \end{array}$$

$$F(s) = \frac{2}{s^3} + \left(-\frac{2}{s^3} - \frac{3}{s^2}\right) e^{-3s}$$