

# QUIZ

Math 3321

Name: Solutions

February 12, 2008

Question 1. Find the general solution.

$$y'' + 4y' + 13y = 0$$

$$r^2 + 4r + 13 = 0$$

$$r_{1,2} = \frac{-4 \pm \sqrt{16 - 4 \cdot 13}}{2} = \frac{-4 \pm \sqrt{16 - 52}}{2}$$

$$= -2 \pm 3i$$

General solutions:

$$y = C_1 e^{-2x} \cos 3x + C_2 e^{-2x} \sin 3x$$

Question 2. What is the differential equation whose fundamental set is

$$\{e^{-x}, e^{2x}\}?$$

Roots  $r = -1$       $r = 2$

$$(r+1)(r-2) = 0$$

$$r^2 - r - 2 = 0$$

Diff. equation:

$$y'' - y' - 2y = 0$$

Question 3. Find the solution of the initial-value problem

$$\begin{cases} y'' + 2y' + y = 0, \\ y(0) = -3, \\ y'(0) = 1. \end{cases}$$

$$r^2 + 2r + 1 = 0$$

$$(r+1)^2 = 0 \quad r = -1 \text{ double root}$$

General solution:

$$y = C_1 e^{-x} + C_2 x e^{-x}$$

$$y' = -C_1 e^{-x} + C_2 e^{-x} - C_2 x e^{-x}$$

Initial conditions:

$$\begin{cases} -3 = C_1 + C_2 \cdot 0 = C_1 \\ 1 = -C_1 + C_2 - C_2 \cdot 0 = -C_1 + C_2 \end{cases}$$

$$\boxed{C_1 = -3}$$

$$1 = -C_1 + C_2 = 3 + C_2$$

$$\boxed{C_2 = -2}$$

Solution of IVP:

$$\boxed{y = -3e^{-x} - 2xe^{-x}}$$