

Name \_\_\_\_\_, \_\_\_\_\_  
Last First

PeopleSoft ID \_\_\_\_\_

### *Instructions*

- This exam consists of 5 work out problems and 1 bonus problem.
- This exam is copied double-sided. Please remember to work all the problems on both sides of each page.
- Show all of your work.
- Be sure to read the instructions to each problem carefully.
- No books nor calculators.
- Single-sided 8.5"×11" page of handwritten notes allowed and must be turned in with the exam.

Problems 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	
Bonus Problem	
Total	

1. (15pts) Three standard fair coins are tossed. A payoff, denote  $X$ , is given as follows:

Win \$1 for each head.

Lose \$2 for each tail.

- (a) What is the sample space for this experiment?
- (b) Determine the probability mass function for  $X$ .
- (c) What is the expected value of  $X$ ?

2. (20pts) Two dice are tossed. Let  $X_1$  and  $X_2$  be defined as follows:

$$X_1 = \begin{cases} 1 & \text{if the first die lands on even numbers} \\ 0 & \text{if the first die lands on odd numbers} \end{cases}$$

$$X_2 = \begin{cases} 1 & \text{if the summation is an even number} \\ 0 & \text{if the summation is an odd number} \end{cases}$$

- (a) Construct a contingency table for the joint probability mass function of  $X = (X_1, X_2)$ .
- (b) What are the marginal probability mass functions of  $X_1$  and  $X_2$ ?
- (c) Are  $X_1$  and  $X_2$  independent? Are they identically distributed?

3. (20pts) Two dice are thrown. Let  $E$  be the event that the sum is 5; let  $F$  be the event that at least one of the dice lands on 1; let  $G$  be the event that the sum is odd.
- (a) How many possible outcomes are there in the sample space  $S$ ?
  - (b) List all the elements in the events  $E \cap F$ ,  $E \cup F$ ,  $E \cap F^c$ ,  $E \cap G$ ,  $G \cap F$ , and  $E \cap F \cap G$  respectively.
  - (c) What are the probabilities of the above 6 events?

4. (20pts) At a psychiatric clinic the social workers are so busy that, on the average, only 70% of potential new patients that telephone are able to talk immediately with a social worker when they call. The other 30% are asked to leave their phone numbers. About  $\frac{2}{3}$  of the time a social worker is able to return the call on the same day, and the other  $\frac{1}{3}$  of the time the caller is contacted on the following day. Experience at the clinic indicates that the probability a caller will actually visit the clinic for consultation is 0.8 if the caller was immediately able to speak to a social worker, whereas it is 0.5 and 0.3, respectively, if the patient's call was returned the same day or the following day.
- (a) What percentage of people that telephone visit the clinic for consultation?
  - (b) What percentage of patients that visit the clinic did not have to have their telephone calls returned?

5. (25pts) A particular casualty insurance company specializes in homeowner's insurance, automobile insurance, and professional liability insurance. Of its customers, 60% have a homeowner's policy issued by the company, 50% have an automobile policy, and 40% have a professional liability policy. Of its customers, 40% have both homeowner's and auto insurance, 25% have both homeowner's and professional liability insurance, and 15% have both auto and professional liability insurance. There are 20% of its customers that do not have any of these types of insurance.
- (a) What fraction of this company's customers have all three types of insurance?
  - (b) What fraction of the company's customers have only professional liability insurance?
  - (c) What fraction of customers with professional liability insurance also have homeowner's insurance?
  - (d) What fraction of customers with auto insurance also have homeowner's insurance?

6. (**BONUS , 5 extra points**)A hand of four cards is dealt from a 52-card deck. What is the probability that the hand contains at least one king?