MATH 3338 Mock Test 3

1. The random variable X has a Gamma distribution with parameters $r = 5$ and $\lambda = 0.5$. Calculate the probability $\Pr(X > 10)$. (Use the table of the incomplete Gamma function.)

2. The moment generating function $M_X(t)$ for a random variable X is

$$M_X(t) = \exp(3t + t^2/8).$$

   (a) Identify the distribution.

   (b) Determine the mean and variance of X.

   (c) Calculate $\Pr(X > 4)$ and $\Pr(-3 < X < 3)$. (Use the table of the Incomplete Gamma Function or the table of the Normal Distribution if needed.)

3. A service center on a major highway has two automated machines located next to each other. Customers wishing to obtain cash from one of these machines form a single queue and use the first machine to become available. On average, the service time at the machine on the left is 30 seconds, and the service time at the machine on the right (a newer model) is 20 seconds. The machines operate on separate power supplies and function independently. Suppose that both machines are currently in use.

   (a) How long should the person at the front of the line expect to wait before a machine becomes available?

   (b) What is the probability that the person at the front of the line will have to wait more than 15 seconds for a machine to become available?

4. Jill and Greg are friendly competitors in high school, and are about to take the ACT exam. Suppose that they have equal ability, and that their ACT score performances may be assumed to be normally distributed with mean 24 and standard deviation 2. What is the probability that their scores differ by more than 5 points in either direction? [15]

5. X and Y are two random variables with joint p.m.f. given by $f(0,0) = f(1,2) = 0.2$, $f(0,1) = f(1,1) = 0.3$.

   (a) Construct the contingency table, and write in the marginal p.m.f.'s. [5]

   (b) Compute $E(X), E(Y), \sigma_X^2, \sigma_Y^2, \text{Cov}(X,Y)$, and the correlation coefficient $\rho_{X,Y}$. [20]