NAME: ____________________________

Department of Mathematics, University of Houston
Math 3333 - Intermediate Analysis - David Blecher
Test 3—August 2010

Instructions. Show all working and reasoning, the points are almost all for logical, complete reasoning. If you use a result from the class notes, state it, but you need not prove it unless you are asked to. [Approximate point values in parentheses, total = 100 points, but there are 5 bonus points]

1. If f is integrable on $[a, b]$ then does it follow that f is differentiable on (a, b)? Prove it or give a counterexample (and short explanation why your example fits the requirements). [7]

2. If $f(x)$ is differentiable at c, prove that $f(x)$ is continuous at c. [7]

3. Let $f(x) = x^2|x|$. Prove that f is differentiable at every point. [9]

4. (a) State and prove Rolle's theorem. [6+9]
   (b) State the mean value theorem. [6]
   (c) Prove that if $f'(x) \leq 0$ for all $x \in (a, b)$ then $f(x)$ is decreasing on (a, b). [7]

5. (a) What can you say about a function $f : (a, b) \to \mathbb{R}$ which is one-to-one and continuous? List as many consequences as you know. [6]
   (b) State the inverse function theorem. [9]

6. (a) Define a partition P of $[a, b]$, define $L(f, P)$ and the lower integral $L(f)$. [10]
   (b) What does it mean to say that a bounded function $f : [a, b] \to \mathbb{R}$ is integrable? [2]
   (c) Give an example of a bounded function on $[0, 1]$ that is not integrable (you need not explain why). [3]

7. (a) If f and g are integrable on $[a, b]$, and $g \le f$ on $[a, b]$, prove that $\int_a^b g\,dx \le \int_a^b f\,dx$. [8]
   (b) Suppose that $f(x) \ge 0$ for all $x \in [a, b]$, and that f is continuous on $[a, b]$, and $\int_a^b f\,dx = 0$. Prove that f is always 0 on $[a, b]$. (Hint: you could use (a), taking g as in picture.) [14]