NAME: ____________________________ Student #: ________________________

Department of Mathematics, University of Houston
Math 3333 - Intermediate Analysis - David Blecher
Mock Test 1.

Instructions. Show all working and reasoning, the points are almost all for logical, complete reasoning. [Approximate point values are given, total ¿ 100 points].

1. What is the negation of the following statement: $\forall\, x \in A$, $\exists y \in B$ such that $x < y < 1$. [5]

2. Prove by mathematical induction: $1 + 3 + 5 + \cdots (2n - 1) = n^2$. [10]

3. Prove that for real numbers, if $x < y + \epsilon\ \forall \epsilon > 0$, then $x \leq y$. [15]

4. (a) What does the Archimidean property state? Also state another fact which also goes by this name. [6]
   (b) Use the Archimidean property to show that $\sup\{n/(n+1) : n \in \mathbb{N}\} = 1$. Include all reasoning. [10]

5. Prove that between any two real numbers, there is a rational number. [15]

6. (a) What is a 'boundary point' of a set S? [4]
   (b) Let S be the set $\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}, \cdots, \frac{999}{1000}\}$. Is 1 a boundary point of S? Prove it. [5]
   (c) Is the set S in (b) closed? Explain. [5]
   (d) Define in terms of boundary points what it means for a set to be open. [4]
   (e) Prove that a set is open (in the sense of (d)) if every number in S is an interior point of S. [6]

7. (a) Give as many alternative descriptions as you can of compact sets. [5]
   (b) State the nested intervals theorem. [6]
   (c) Prove that if S is a nonempty set which is compact then S has a maximum. [6]