Department of Mathematics, University of Houston
Math 4332. Intro to Real Analysis. David Blecher, Spring 2015
Homework 13.

As usual, exercises marked with * are to be turned in by the graduate students in the class.

(1) Let $f(u,v) = (u^3 - v^2, \sin u - \ln v)$ and $\vec{x}_0 = (-1,0)$. Show that f^{-1} exists and is differentiable on an open neighborhood of $\vec{x}_0$, and compute $(f^{-1})'(\vec{x}_0)$.

(2) Consider the system

$$\begin{cases} u = x + xyz \\ v = y + xy \\ w = z + 2x + 3z^2. \end{cases}$$

(i) Show using the inverse function theorem that it is possible to solve for x, y, and z above explicitly in terms of u, v, w, in a neighborhood of the point $(x,y,z) = (0,0,0)$.
(ii) Using the inverse function theorem find the Jacobian matrix of derivatives of x, y, and z with respect to u, v, w, at the point $(0,0,0)$.
(iii) Find the first partial derivatives of x with respect to u, v, w, at the point where $(x,y,z) = (0,0,0)$.

(3) Let $f(x,y) = 2x^3 - 3x^2 + 2y^3 + 3y^2$. Set $Z = \{(x,y) \in \mathbb{R}^2 : f(x,y) = 0\}$. Find those points of Z which have no neighborhoods in which the equation $f(x,y) = 0$ can be solved explicitly for y in terms of x.

(4) Prove that there are functions $u(x,y), v(x,y)$, and $w(x,y)$, and an $r > 0$ such that u, v, w are C^1 on $B((1,1), r)$, satisfy $u(1,1) = 1, v(1,1) = 1, w(1,1) = -1$, and satisfy the equations

$$\begin{cases} u^5 + xv^2 - y + w = 0 \\ v^5 + yu^2 - x + w = 0 \\ w^4 + y^5 - x^4 = 1. \end{cases}$$

(5) Define $f(x, y_1, y_2) = x^2 y_1 + e^x + y_2$. Show that $f(0,1,-1) = 0$ and that there exists a differentiable function g in an open set containing $(1,-1)$ in $\mathbb{R}^2$, such that $g(1,-1) = 0$ and $f(g(y_1,y_2), y_1, y_2) = 0$. Find $g'(1,-1)$.

(6*) If $S \subseteq \mathbb{R}^n$ is open, if $f : S \to \mathbb{R}$ has partial derivatives at all points in S, and if there is a constant $M > 0$ with $|\frac{\partial f}{\partial x_i}(\vec{x})| \leq M$ for all $\vec{x} \in S$, $i = 1, \ldots n$, then show f is continuous on S.

(7) For extra practice you could do some of Problems 2.30, 2.37 (see question 4 above), and 2.38, in Paulsen's notes.