Department of Mathematics, University of Houston
Math 4332. Intro to Real Analysis. David Blecher, Spring 2015
Homework 9.

As usual, exercises marked with * are to be turned in by the graduate students in the class.

(1) Using the elementary function definitions from class, prove that $e^{iz} = \cos z + i \sin z$, and that $\cos x = 0$ has a real solution x. Also show that the function $\gamma(\theta) = e^{i\theta}$ from $[0, 2\pi)$ to the unit circle in the complex plane (which one may identify with the map $\gamma(\theta) = (\cos\theta, \sin\theta)$ into the unit circle in $\mathbb{R}^2$) is one-to-one and onto the unit circle. [

(2) If f is a continuous scalar valued function on $[a, b]$ and $f(a) = 0$, prove that there is a sequence of polynomials p_n with $p_n(a) = 0$ for all n, and $p_n \stackrel{u}{\to} f$ on $[a, b]$.

(3) Problems 1.80 and 1.82* in the notes.

(4) For f, g as in our discussion of convolution on $\mathbb{R}$, show that $f * g$ is continuous (grad students only), and that $f * (g + ch) = f * g + c(f * h)$, for constant c.

(5) Explain in detail why the 'rephrased versions' of the Weierstrass theorem stated in class are all equivalent.

(6) Look at the Stone-Weierstrass theorem and its proof in the notes. Then do Problem 1.89 in the notes. Also explain in detail why the Stone-Weierstrass theorem immediately implies the Weierstrass polynomial approximation theorem done in class.