Department of Mathematics, University of Houston
Math 6342–Topology. David Blecher
Homework 3

Instructions. You may ignore problems you have done before in previous classes (which you remember how to do). You are encouraged to work with others, form study groups, and so on; however do not copy homework you are required to turn in.

1. Consider the set $Y = [-1, 1]$ as a subspace of $\mathbb{R}$. Which of the following sets are open in Y? Which are open in $\mathbb{R}$?
$$A = \{x : \frac{1}{2} < |x| < 1\}, B = \{x : \frac{1}{2} < |x| \leq 1\}, C = \{x : \frac{1}{2} \leq |x| < 1\},$$
$$D = \{x : \frac{1}{2} \leq |x| \leq 1\}, E = \{x : 0 < |x| < 1 \text{ and } \frac{1}{x} \notin \mathbb{Z}_+\},$$
2. Show that if A is relatively closed in Y and Y is closed in X, then A is closed in X.
3. Show that if U is open in X and A is closed in X, then $U \setminus A$ is open in X, and $A \setminus U$ is closed in X.
4. Let A_α denote subsets of a space X. Prove that $\bigcup \overline{A_\alpha} \subseteq \overline{\bigcup A_\alpha}$. Give an example where equality fails.
5. Criticize the following "proof" that $\overline{\bigcup A_\alpha} \subseteq \bigcup \overline{A_\alpha}$: if $\{A_\alpha\}$ is a collection of sets in X and if $x \in \overline{\bigcup A_\alpha}$, then every neighborhood N of x intersects $\bigcup A_\alpha$. Thus N must intersect some A_α, so that x must belong to the closure of some A_α. Therefore, $x \in \bigcup \overline{A_\alpha}$.
6. Prove the variant of Proposition 1.2.9, but for A' in place of ∂A. [Hint: The only parts of this Proposition which remain correct are the results called Theorem 17.6 and Corollary 17.7 in the text. The proofs may be found there.]