Department of Mathematics, University of Houston
Math 6342. Topology. David Blecher
Homework 9 (Algebraic Topology)

1. If X is path connected, and $\hat{\beta}$ is as defined in 3.2.8, show that $\Pi(X, x_0)$ is abelian iff $\hat{\alpha} = \hat{\beta}$ for every pair of paths α, β from x_0 to another point $x_1 \in X$.

2. Let $h : X \to Y$ be continuous, with $h(x_0) = y_0$ and $h(x_1) = y_1$. Let α be a path in X from x_0 to x_1, and let $\beta = h \circ \alpha$. Show that $\widehat{\beta} \circ (h_{x_0})_* = (h_{x_1})_* \circ \widehat{\alpha}$.

3. If $A \subset X$ then a retraction of X onto A is a continuous $r : X \to A$ with $r(a) = a$ for $a \in A$. If $a_0 \in A$ prove that $r_* : \pi_1(X, a_0) \to \pi_1(A, a_0)$ is a surjection (onto) between these fundamental groups.

4. Let $p : E \to B$ be continuous and surjective. Suppose that U is an open set of B that is evenly covered by p. Show that if U is connected, then the partition of $p^{-1}(U)$ into slices is unique.

5. Let $p : E \to B$ be a covering map ; let B be connected. Show that if $p^{-1}(\{b_0\})$ has k elements for some $b_0 \in B$, then $p^{-1}(\{b\})$ has k elements for every $b \in B$.

6. Let $p : E \to B$ be a covering map with E path connected. Show that if B is simply connected, then p is a homeomorphism.

7. In class we briefly discussed the annulus, $\bar{B}((0,0),2) \setminus B((0,0),1)$, a closed disk with a concentric open disk removed. Show that a covering space for this is a closed infinite strip $R \times [1,2]$, and using this and the lifting correspondence theorem show that the fundamental group of the annulus is the integers.

8. Let $p : E \to B$ be a covering map. Show: (a) If B is Hausdorff, regular, completely regular, or locally compact Hausdorff, then so is E. (b) If B is compact and $p^{-1}(\{b\})$ is finite for each $b \in B$, then E is compact.