1432 Review – Test 3

1. Integrate:

   a. $\int \frac{x+2}{x+1}\,dx$

   b. $\int \frac{3x^2+3x+3}{x^2+1}\,dx$

   c. $\int_0^{\frac{\sqrt{3}}{2}} \frac{1}{\sqrt{1-x^2}}\,dx$

   d. $\int \frac{x^2}{(x+1)(x-1)^2}\,dx$

   e. $\int \frac{x^2+5x+2}{(x+1)(x^2+1)}\,dx$

   f. $\int \frac{2x^2}{\sqrt{9-x^2}}\,dx$

   g. $\int \frac{2}{x\sqrt{9+x^2}}\,dx$

2. Given $\int_1^5 (2x+1)\,dx$

   Use the trapezoid method with n = 4 to approximate the integral.
   Use the midpoint method with n = 4 to approximate the integral.
   Use Simpson's rule with n = 4 to approximate the integral.

3. Determine the values of n which guarantee a theoretical error less than ε if the integral is estimated by the trapezoidal rule if ε = 0.01.

   $\int_1^3 \left(\frac{1}{4}x^2 + 3x - 2\right) dx$

4. Plot the following polar coordinates:
   a. $[3, \pi]$
   b. $\left[-2, \frac{2}{3}\pi\right]$

5. Write the equation in polar coordinates:
   a. $x^2 + y^2 = 4$
   b. $x^2 + y^2 = 4x$
   c. $\left(x^2 + y^2\right)^2 = 4xy$
   d. $x = 4y$

6. Write the given equations in rectangular coordinates:
   a. $r = -2\sin\theta$
   b. $r\cos\theta = 5$

7. Given $r = 4 - 8\cos\theta$, give the formula (only) for the area inside the inner loop.

8. Given $r = 2\sin(3\theta)$, give the formula (only) for the area of one petal.

9. Find the arc length for the following:
   a. $f(x) = \frac{2}{3}(x-1)^{3/2} \quad x \in [1,2]$
   b. $x(t) = \sin(2t),\ y(t) = \cos(2t), \quad t \in \left[0, \frac{\pi}{2}\right]$
   c. $r = 2\sec(\theta), \quad t \in \left[0, \frac{\pi}{4}\right]$

10. Find the equation of the tangent and the normal lines to the parametric curves at the given points:

a. $x(t) = -2\cos 2t,\ y(t) = 4 + 2t,\ (-2,4)$

b. $x(t) = 3\cos(3t) + 2t,\ y(t) = 1 + 5t,\ (3,1)$

8. Find the points (x, y) at which the curve $x(t) = 3 - 4\sin(t), \quad y(t) = 4 + 3\cos(t)$ has:
   (a) a horizontal tangent; (b) a vertical tangent.

9. Give an equation relating x and y for the curve given parametrically by

a. $x(t) = -1 + 3\cos t \quad y(t) = 1 + 2\sin t$

b. $x(t) = -1 + 3\cosh t \quad y(t) = 1 + 2\sinh t$

c. $x(t) = -1 + 4e^t \quad y(t) = 2 + 3e^{-t}$

10. Find a parameterization for:

    a. Line segment from (-1, 3) to (5, 4)

    b. Circle with radius 2 and center (2, -1)

11. Write an expression for the nth term of the sequence:

    a. 1, 4, 7, 10, …

    b. 2, -1, $\frac{1}{2},-\frac{1}{4},\frac{1}{8},\ldots.$

12. Determine if the following sequences are monotonic. Also indicate if the sequence is bounded and if it is give the least upper bound and/or greatest lower bound.

    a. $a_n = \frac{2n}{1+n}$

    b. $a_n = \frac{\cos n}{n}$

13. Determine if the following sequences converge or diverge. If they converge, give the limit.

    a. $\left\{(-1)^n\left(\frac{n}{n+1}\right)\right\}$

    b. $\left\{\frac{6n^2-2n+1}{4n^2-1}\right\}$

    c. $\left\{\frac{(n+2)!}{n!}\right\}$

    d. $\left\{\frac{3}{e^n}\right\}$

    e. $\left\{\frac{4n+1}{n^2-3n}\right\}$

    f. $\left\{\frac{e^n}{n^3}\right\}$