NAME: _____

Recitation Instructor: _____

Part I. Techniques of Integration

1. $\displaystyle\int \frac{1}{x^2\sqrt{1-x^2}}\,dx$ **Answer:** _____

2. $\displaystyle\int \frac{1}{x^4-16}\,dx$ **Answer:** _____

3. $\displaystyle\int \frac{x^2+8x-3}{x^3+3x^2}\,dx$ **Answer:** _____

4. $\displaystyle\int \frac{x}{\sqrt{4+x^2}}\,dx$ **Answer:** _____

5. $\displaystyle\int_0^1 \frac{x^2}{(4-x^2)^{3/2}}\,dx =$ **Answer:** _____

6. $\displaystyle\int \frac{x^2+2x-4}{x^3-4x}\,dx$ **Answer:** _____

Part II. Numerical Integration

Set $f(x) = x^2 + 1$ on $[0, 4]$.

1. Use the midpoint rule with $n = 4$ to approximate $\displaystyle\int_0^4 f(x)\,dx$.

Answer: _____

2. Use the trapezoidal rule with $n = 4$ to approximate $\displaystyle\int_0^4 f(x)\,dx$.

Answer: _____

3. Use Simpson's rule with $n = 2$ to approximate $\displaystyle\int_0^4 f(x)\,dx$.

Answer: _____

4. Determine the smallest integer n such that the trapezoidal approximation T_n approximates $\int_0^4 f(x)\,dx$ with error less than 0.0075.

Answer: _____

5. Determine the error if S_4 is used to estimate $\int_0^2 e^x\,dx$. (Use $e \approx 3$)

Answer: _____

Part III. Polar Coordinates

1. Give the rectangular coordinates of the point with polar coordinates $[-2, 8\pi/3]$.

Answer: _____

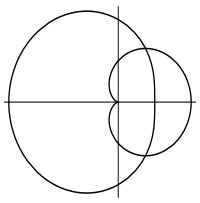
2. Give all possible polar coordinates for the point with rectangular coordinates $(-4\sqrt{3}, 4)$.

Answer: _____

3. Sketch the graph of $r = 1 + 2\cos\theta, \quad 0 \le \theta \le 4\pi/3$.

Answer: _____

The graphs of $C_1: \ r = 2 - \cos\theta$ and $C_2: \ r = 1 + \cos\theta$ are shown in the figure.



4. Calculate the area of the region inside C_2 and outside C_1.

Answer: _____

5. Calculate the area of the region common to C_1 and C_2.

Answer: _____

6. Find the polar equation for $(x^2 + y^2)^2 = 4xy$.

Answer: _____

7. Write the equation $r = 4 \sin \theta$ in rectangular coordinates.

Answer: _____

Part IV. Parametric Equations

1. Express the curve $x = 2 + \sin t,\ y = -1 + \cos t$ by an equation in x and y.

Answer: _____

2. Find a parametrization of the line segment from $(-2, 3)$ to $(1, 5)$.

Answer: _____

3. Find a parametrization for the curve $y^3 = x^2$ from $(1, 1)$ to $(8, 4)$.

Answer: _____

4. Give an equation for the normal line to the graph of $x = \sin t,\ y = 2 + \cos 2t$ at the point where $t = \pi/6$.

Answer: _____

5. Give an equation for the line tangent to the polar curve $r = 2 \cos \theta$ at the point where $\theta = \pi/3$

Answer: _____

6. Find the points (x, y) at which the curve $x = t^2 - 2t,\ y = \frac{1}{3}t^3 - 3t^2 + 8t$ has (a) a horizontal tangent, (b) a vertical tangent.

Answer: (a) _____

Answer: (b) _____

7. Find the length of the curve $C:\ x = t^2 + 1,\ y = \frac{4}{3}t^3 - 3,\ 0 \le t \le 2.$

Answer: _____

8. Find the length of the polar curve $r = 1 - \cos\theta,\ 0 \le \theta \le 2\pi.$

Answer: _____

9. Find the length of the graph of $f(x) = \frac{1}{3}(x+2)^{3/2},\ 0 \le x \le 2.$

Answer: _____

10. A particle moves along the curve $x = \frac{1}{3}t^3 - t,\ y = t^2 + 2,\ 0 \le t \le 2.$ (a) What is the speed of the particle at time t? (b) What is the total distance traveled by the particle?

Answer: (a)_____

Answer: (b) _____

Part IV. Sequences

1. Determine a formula for a_n, the general term of the given sequence. Then determine whether the sequence converges and if it does, give the limit.

$(a)\quad 4,\ 1,\ \frac{1}{4},\ \frac{1}{16}, \ldots.$ $(b)\quad \frac{2}{1},\ \left(\frac{3}{2}\right)^2,\ \left(\frac{4}{3}\right)^3,\ \left(\frac{5}{4}\right)^4,\ \ldots$

Answer: (a)_____

Answer: (b) _____

2. Determine whether or not the given sequence is bounded above, bounded below, bounded. If it is bounded above or below, give the least upper and/or greatest lower bounds.

$(a)\quad \{\cos(n\pi/3)\}$ $(b)\quad \left\{\frac{n^3+1}{n^2+2n+3}\right\}$ $(c)\quad \left\{2 + \frac{(-1)^n}{n}\right\}$

Answer: (a)_____

Answer: (b) _____

Answer: (c) _____

3. Determine the monotonicity of the given sequence.

(a) $\{(2/3)^n\}$ (b) $\left\{\dfrac{n^2}{n+2}\right\}$ (c) $\left\{\dfrac{n+(-1)^n}{n^2}\right\}$

Answer: (a)_____

Answer: (b) _____

Answer: (c) _____

4. Determine whether or not the given sequence converges or diverges. If it converges, give the limit.

(a) $\left\{\dfrac{n^2+1}{\sqrt{4n^4+2n^2+1}}\right\}$ (b) $\left\{\dfrac{\sin^2 n}{n}\right\}$ (c) $\left\{\dfrac{(-1)^n(2n)}{\sqrt{n^2+4}}\right\}$

Answer: (a)_____

Answer: (b) _____

Answer: (c) _____