Answers, Review Questions, Exam 3

Part I. Techniques of Integration

1. $\displaystyle\int \frac{1}{x^2\sqrt{1-x^2}}\,dx$

Answer: $\displaystyle -\frac{\sqrt{1-x^2}}{x} + C$

2. $\displaystyle\int \frac{1}{x^4-16}\,dx$

Answer: $\frac{1}{32}\ln|x-2| - \frac{1}{32}\ln|x+2| + \frac{1}{16}\tan^{-1}(x/2) + C$

3. $\displaystyle\int \frac{x^2+8x-3}{x^3+3x^2}\,dx$

Answer: $\displaystyle \frac{1}{x} + 3\ln|x| - 2\ln|x+3| + C$

4. $\displaystyle\int \frac{x}{\sqrt{4+x^2}}\,dx$

Answer: $\sqrt{4+x^2} + C$

5. $\displaystyle\int_0^1 \frac{x^2}{(4-x^2)^{3/2}}\,dx =$

Answer: $\displaystyle \frac{1}{\sqrt{3}} - \frac{\pi}{6}$

6. $\displaystyle\int \frac{x^2+2x-4}{x^3-4x}\,dx$

Answer: $\ln|x| + \frac{1}{2}\ln|x-2| - \frac{1}{2}\ln|x+2| + C$

Part II. Numerical Integration

Set $f(x) = x^2 + 1$ on $[0, 4]$.

1. Use the midpoint rule with $n = 4$ to approximate $\displaystyle\int_0^4 f(x)\,dx$.

Answer: 25

2. Use the trapezoidal rule with $n = 4$ to approximate $\displaystyle\int_0^4 f(x)\,dx$.

Answer: 26

3. Use Simpson's rule with $n = 2$ to approximate $\displaystyle\int_0^4 f(x)\,dx$.

Answer: $\dfrac{76}{3}$

4. Determine the smallest integer n such that the trapezoidal approximation T_n approximates $\int_0^4 f(x)\,dx$ with error less than 0.0075.

Answer: 38

5. Determine the error if S_4 is used to estimate $\int_0^2 e^x\,dx$. (Use $e \approx 3$)

Answer: 0.0042

Part III. Polar Coordinates

1. Give the rectangular coordinates of the point with polar coordinates $[-2, 8\pi/3]$.

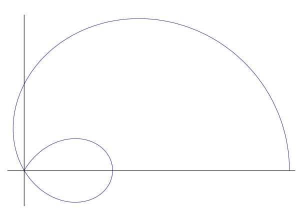
Answer: $(1, -\sqrt{3}\,)$

2. Give all possible polar coordinates for the point with rectangular coordinates $(-4\sqrt{3}, 4)$.

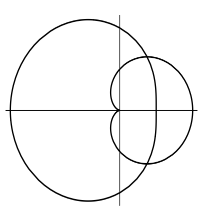
Answer: $[8, \frac{5}{6}\pi + 2n\pi]$, $[-8, \frac{11}{6}\pi + 2n\pi]$

3. Sketch the graph of $r = 1 + 2\cos\theta$, $0 \le \theta \le 4\pi/3$.

Answer:



The graphs of $C_1:\ r = 2 - \cos\theta$ and $C_2:\ r = 1 + \cos\theta$ are shown in the figure.



4. Calculate the area of the region inside C_2 and outside C_1.

Answer: $3\sqrt{3} - \pi$

5. Calculate the area of the region common to C_1 and C_2.

Answer: $\frac{5}{2}\pi - 3\sqrt{3}$

6. Find the polar equation for

$$(x^2 + y^2)^2 = 4xy$$

Answer: $r^2 = 2\sin 2\theta$

7. Write the equation $r = 4\sin\theta$ in rectangular coordinates.

Answer: $x^2 + (y-2)^2 = 4$; a circle: center $(0,2)$, radius 2

Part IV. Parametric Equations

1. Express the curve $x = 2 + \sin t$, $y = -1 + \cos t$ by an equation in x and y.

Answer: $(x-2)^2 + (y+1)^2 = 1$; circle: center $(2,-1)$, radius 1

2. Find a parametrization of the line segment from $(-2,3)$ to $(1,5)$.

Answer: $x = -2 + 3t, \quad y = 3 + 2t, \quad 0 \le t \le 1$

3. Find a parametrization for the curve $y^3 = x^2$ from $(1,1)$ to $(8,4)$.

Answer: $x = t, \quad y = t^{3/2}, \quad 1 \le t \le 8$

4. Give an equation for the normal line to the graph of $x = \sin t$, $y = 2 + \cos 2t$ at the point where $t = \pi/6$.

Answer: $y - \frac{5}{2} = \frac{1}{2}(x - \frac{1}{2})$

5. Give an equation for the line tangent to the polar curve $r = 2\cos\theta$ at the point where $\theta = \pi/3$

Answer: $y - \dfrac{\sqrt{3}}{2} = \dfrac{1}{\sqrt{3}}\left(x - \dfrac{1}{2}\right)$

6. Find the points (x, y) at which the curve $x = t^2 - 2t,\ y = \frac{1}{3}t^3 - 3t^2 + 8t$ has (a) a horizontal tangent, (b) a vertical tangent.

Answer: (a) $\left(8,\, \frac{16}{3}\right),\ \left(0,\, \frac{20}{3}\right)$

Answer: (b) $\left(-1,\, \frac{16}{3}\right)$

7. Find the length of the curve $C:\ x = t^2 + 1,\ y = \frac{4}{3}t^3 - 3,\ 0 \le t \le 2.$

Answer: $\frac{1}{6}(17)^{3/2} - \frac{1}{6}$

8. Find the length of the polar curve $r = 1 - \cos\,\theta,\ 0 \le \theta \le 2\pi.$

Answer: 8

9. Find the length of the graph of $f(x) = \frac{1}{3}(x+2)^{3/2},\ 0 \le x \le 2.$

Answer: $\frac{1}{3}\left[8^{3/2} - 6^{3/2}\right] = \frac{16}{3}\sqrt{2} - 2\sqrt{6}$

10. A particle moves along the curve $x = \frac{1}{3}t^3 - t,\ y = t^2 + 2,\ 0 \le t \le 2.$ (a) What is the speed of the particle at time t? (b) What is the total distance traveled by the particle?

Answer: (a) speed: $t^2 + 1$

Answer: (b) distance traveled: $\frac{14}{3}$

Part IV. Sequences

1. Determine a formula for a_n, the general term of the given sequence. Then determine whether the sequence converges and if it does, give the limit.

$$(a)\quad 4,\ 1,\ \frac{1}{4},\ \frac{1}{16},\ldots.$$

$$(b)\quad \frac{2}{1},\ \left(\frac{3}{2}\right)^2,\ \left(\frac{4}{3}\right)^3,\ \left(\frac{5}{4}\right)^4,\ \ldots$$

Answer: (a) $a_n = 4\left(\frac{1}{4}\right)^{n-1};\quad a_n \to 0$

Answer: (b) $a_n = \left(\frac{n+1}{n}\right)^n = \left(1 + \frac{1}{n}\right)^n$

$\to e$

2. Determine whether or not the given sequence is bounded above, bounded below, bounded. If it is bounded above or below, give the least upper and/or greatest lower bounds.

$$(a)\quad \{\cos(n\pi/3)\} \qquad\qquad (b)\quad \left\{\frac{n^3+1}{n^2+2n+3}\right\}$$

$$(c)\quad \left\{2+\frac{(-1)^n}{n}\right\}$$

Answer: (a) bounded; glb $=-1$, lub $=1$

Answer: (b) not bounded above; bounded below, glb $=\frac{1}{3}$

Answer: (c) bounded; glb $=1$, lub $=\frac{5}{2}$

3. Determine the monotonicity of the given sequence.

$$(a)\quad \{(2/3)^n\} \qquad\qquad (b)\quad \left\{\frac{n^2}{n+2}\right\}$$

$$(c)\quad \left\{\frac{n+(-1)^n}{n^2}\right\}$$

Answer: (a) decreasing

Answer: (b) increasing

Answer: (c) not monotone

4. Determine whether or not the given sequence converges or diverges. If it converges, give the limit.

$$(a)\quad \left\{\frac{n^2+1}{\sqrt{4n^4+2n^2+1}}\right\} \qquad\qquad (b)\quad \left\{\frac{\sin^2 n}{n}\right\}$$

$$(c)\quad \left\{\frac{(-1)^n(2n)}{\sqrt{n^2+4}}\right\}$$

Answer: (a) converges, $a_n \to \frac{1}{2}$

Answer: (b) converges, $a_n \to 0$

Answer: (c) diverges