L'Hôpital's Rule: In each of the following, determine whether or not the given limit is an indeterminate form. If it is an indeterminate form, give the type (e.g., $0/0$, ∞/∞, $0 \cdot \infty$ etc.). Evaluate each of the limits.

(a) $\displaystyle\lim_{x\to 0} \frac{1+x-e^x}{x^2}$ (b) $\displaystyle\lim_{x\to 1} \frac{x+\ln x}{2x^2}$ (c) $\displaystyle\lim_{x\to \pi/2} \left(x-\frac{\pi}{2}\right)\tan x$

(d) $\displaystyle\lim_{x\to\infty} \left(1+\frac{2}{x}\right)^{2x}$ (e) $\displaystyle\lim_{x\to 0} \frac{1-\cos x}{x^2}$ (f) $\displaystyle\lim_{x\to 0^+} \left(\frac{1}{x}-\frac{1}{\sqrt{x}}\right)$

Improper Integrals: Determine whether or not the given integral is improper. If it is improper, determine whether it converges or diverges. In the case of convergence, give the value.

(a) $\displaystyle\int_2^6 \frac{1}{\sqrt{x-2}}\,dx$ (b) $\displaystyle\int_{-1}^1 \frac{1}{4-x^2}\,dx$ (c) $\displaystyle\int_0^\infty x\,e^{-x}\,dx$

(d) $\displaystyle\int_{-\infty}^\infty \frac{1}{1+x^2}\,dx$ (e) $\displaystyle\int_1^\infty \frac{1}{\sqrt{x-1}}\,dx$

Infinite Series, General:

(a) The series $4-3+\frac{9}{4}-\frac{27}{16}+\cdots$ is a geometric series. Find the general term a_k and write the series in the form $\displaystyle\sum_{k=0}^\infty a_k$. does the series converge? If so, what is the sum?

(b) Given the series: $\sin(\pi/2)+\sin(3\pi/2)+\sin(5\pi/2)+\cdots = \displaystyle\sum_{k=0}^\infty \sin\left[(2k+1)\pi/2\right]$.

Find the general term s_n for the sequence of partial sums of the series. Does the series converge or diverge?

(c) Given the series $2-\frac{3}{2}+\frac{4}{3}-\frac{5}{4}+\cdots$. What is the general term a_k of the series? Determine whether the series converges or diverges.

Nonnegative Series: Determine whether the given series converges or diverges; state which test you are using to determine convergence/divergence and show all work.

(a) $\displaystyle\sum_{k=0}^\infty \frac{k^2\,2^k}{(k+1)!}$ (b) $\displaystyle\sum_{k=0}^\infty \frac{3^{k+1}}{(k+1)^2\,e^k}$

(c) $\displaystyle\sum_{k=1}^\infty \frac{\ln k}{k}$ (d) $\displaystyle\sum_{k=0}^\infty \frac{2k+1}{\sqrt{k^5+3k^3+4}}$

Arbitrary Series:

(a) In each of the following, determine whether the given series is absolutely convergent, conditionally convergent, or divergent; state which test you are using to determine convergence/divergence, and show all work.

(*i*) $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k}{\sqrt{k^2+3k+2}}$ (*ii*) $\displaystyle\sum_{k=0}^{\infty} \frac{\sin(k)}{(k+1)^2}$ (*iii*) $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k\, k^2}{2^k}$

(b) Does the series $\displaystyle\sum_{k=0}^{\infty} \frac{k^2}{(\ln 2)^k}$ converge or diverge? Does the series $\displaystyle\sum_{k=0}^{\infty} \frac{k^3}{(\ln 3)^k}$ converge or diverge?

Power Series:

Find the radius of convergence and the interval of convergence of each of the following power series

(a) $\displaystyle\sum_{k=2}^{\infty} \frac{(-1)^k}{4^k\, \ln\, k}\, x^k$ (b) $\displaystyle\sum_{k=0}^{\infty} \frac{1}{k^3+1}\, x^k$

(c) $\displaystyle\sum_{k=0}^{\infty} \frac{1}{(k+1)3^k}\, (x+1)^k$ (d) $\displaystyle\sum_{k=0}^{\infty} \frac{(-2)^k}{\sqrt{k+1}}\, x^k$

(e) $\displaystyle\sum_{k=0}^{\infty} \frac{k!}{4^k}\, (x-3)^k$ (f) $\displaystyle\sum_{k=0}^{\infty} \frac{k}{k^3+2}\, x^k$

Taylor Polynomials, Taylor Series:

(a) Determine the Taylor polynomial in powers of x of degree 4 for the function $f(x) = (1+2x)^{3/2}$.

(b) Use the Taylor series expansion (in powers of x) for $f(x) = e^x$ to find the Taylor series expansion of $g(x) = \cosh x$.

(c) Determine the Taylor polynomial in powers of $x - \pi/6$ of degree 5 for $f(x) = \sin x$.

(d) The Taylor series expansion of $f(x) = \dfrac{x}{1+2x}$ is:

$$f(x) = x - 2x^2 + 4x^3 - 8x^4 + \cdots = \sum_{k=1}^{\infty} (-1)^{k-1} 2^{k-1}\, x^k.$$

What is $f^{(9)}(0)$?

Approximation by Taylor Polynomials:

(a) Assume that f is a function such that $\left|f^{(n)}(x)\right| \leq 2$ for all n and x.

(*i*) Estimate the error if $P_4(0.5)$ is used to approximate $f(0.5)$.

(*ii*) Find the least integer n for which $P_n(0.5)$ approximates $f(0.5)$ correct to four decimal places.

(b) Let $f(x) = e^x$.

(*i*) Estimate the error if $P_4(0.5)$ is used to approximate $\sqrt{e}$.

(ii) Find the least integer n for which $P_n(0.5)$ approximates $\sqrt{e}$ with error less than 10^{-4}.

(c) Let $f(x) = \cos x$.

(i) Estimate the error if P_2 in powers of $x - \pi/4$ is used to approximate $\cos 40°$.

(ii) Find the least integer n for which P_n approximates $\cos 40°$ with 5 decimal place accuracy.