

MATH 3321(H)

Homework Assignment #1

Due date: Monday, September 1, 2014

NAME (Print): _____

PeopleSoft ID: _____

Instructions:

- Print out this file and complete all the problems.
 - Write your solutions in the space provided.
 - Use a blue or black pen, or a pencil. Xerox copies of your homework are not acceptable.
 - Your homework must be complete, neatly written and stapled.
 - Submit your completed homework in class on the due date.
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1. Given the differential equation $y'' + y = 3 \cos 2x$. Determine whether or not

$$y_1 = 2 \sin x - 3 \cos 2x, \quad y_2 = 3 \sin x - 2 \cos 2x$$

are solutions

2. Given the differential equation $x^2y'' - 3xy' + 4y = 0$. Determine whether or not

$$y_1 = x^2, \quad y_2 = x^2 \ln x$$

are solutions.

3. Given the differential equation $y' + 5y = 2$.

(a) Show that $y = \frac{2}{5} + Ce^{-5x}$ is a solution, where C is any constant.

(b) Assuming that every solution has this form, find a solution that satisfies $y(1) = 2$.

4. Given the differential equation $y'' - 4y = 0$.

(a) Show that $y = C_1 e^{2x} + C_2 e^{-2x}$ (C_1, C_2 any constants) is a solution.

(b) Show that $y = C_1 \cosh 2x + C_2 \sinh 2x$ (C_1, C_2 any constants) is a solution.

(c) Are the families in (a) and (b) different, or are they, in fact, the same family? Explain

5. Find the values of m , if any, such that $y = e^{mx}$ is a solution of

$$y'' - 2y' - 8y = 0.$$

6. Find the values of m , if any, such that $y = e^{mx}$ is a solution of

$$y''' - 3y'' - 4y' + 12y = 0.$$

7. Find the values of r , if any, such that $y = x^r$ is a solution of

$$\frac{d^2y}{dx^2} - \frac{1}{x} \frac{dy}{dx} - \frac{8}{x^2} y = 0.$$

8. Find the general solution of $(1 + x^2)y' = x$.

9. Find the general solution of $y' + (\tan x)y = 0$.

10. Find the differential equation that has $y^2 = x^2 - Cx$ as its general solution.

11. Find the differential equation that has $y = C_1e^{2x} + C_2e^{-3x}$ as its general solution.

12. Find the differential equation that has $y = C_1 + C_2x^3$ as its general solution.

13. Given that every solution of $y'' - y' - 12y = 0$ is a member of the two-parameter family $y = C_1e^{4x} + C_2e^{-3x}$, find a solution of the initial-value problem:

(a) $y'' - y' - 12y = 0, \quad y(0) = 5, \quad y'(0) = 6.$

(b) $y'' - y' - 12y = 0, \quad y(0) = -2, \quad y'(0) = 6.$

14. Given that every solution of $y'' + y = 0$ is a member of the two-parameter family $y = C_1 \cos x + C_2 \sin x$, show that:

(a) the boundary problem $y'' + y = 0$, $y(0) = 0$, $y(\pi/2) = 1$ has a unique solution.

(b) the boundary problem $y'' + y = 0$, $y(0) = 0$, $y(\pi) = 0$ has infinitely many solutions.

(c) the boundary problem $y'' + y = 0$, $y(0) = 1$, $y'(\pi/2) = 1$ does not have a solution.

15. Is the ODE $y'' + xy'y' + y = 2$ linear or nonlinear? How about the DE $(y')^2 - xy' + y = 0$.

16. Solve the ODE $(y')^2 - xy' + y = 0$ (Hints: Differentiate the equation. There is a singular solution).