Name: _____
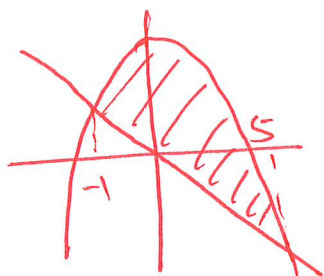
Ps ID: _____

Each question is worth 2 points.

1. Find the average value of $f(x) = 4x^3 - 6x$ over the interval $[-1, 2]$.

$$A.V. = \frac{1}{2-(-1)} \int_{-1}^{2} (4x^3 - 6x)\,dx = \frac{1}{3}(x^4 - 3x^2)\Big|_{-1}^{2} = \frac{1}{3}\left[(16-12) - (1-3)\right]$$
$$= \frac{1}{3}[4+2] = 2$$

2. **Find** the area of the region enclosed by the graphs of $f(x) = 5 - x^2$ and $y = -4x$. (Graph the region.)

$$5 - x^2 = -4x$$
$$x^2 - 4x + 5 = 0$$
$$(x-5)(x+1) \geq 0$$
$$x = 5, -1$$

$$Area = \int_{-1}^{5} (5-x^2) - (-4x)\,dx$$
$$= \int_{-1}^{5} 5 - x^2 + 4x\,dx$$
$$= 5x - \frac{x^3}{3} + 2x^2\Big|_{-1}^{5}$$
$$= (25 - \frac{125}{3} + 50) - (-5 + \frac{1}{3} + 2)$$
$$= 75 - \frac{125}{3} + 3 - \frac{1}{3} = 78 - \frac{126}{3}$$
$$= 78 - 42 = 36$$

3. Evaluate: $\int \frac{4x+2}{x^2+9}\,dx$

$u = x^2 + 9$
$du = 2x\,dx$

$$= 4\int \frac{x}{x^2+9}\,dx + 2\int \frac{dx}{x^2+9}$$
$$= 2\int \frac{du}{u} + 2\int \frac{dx}{x^2+9}$$
$$= 2\ln|u| + \frac{2}{3}\tan^{-1}\frac{x}{3} + C = 2\ln|x^2+9| + \frac{2}{3}\tan^{-1}\frac{x}{3} + C$$

4. Evaluate: $\int x^2\sqrt{x^3+1}\,dx$

$u = x^3 + 1$
$du = 3x^2\,dx$
$\frac{du}{3} = x^2\,dx$

$$= \frac{1}{3}\int \sqrt{u}\,du$$
$$= \frac{1}{3}\int u^{1/2}\,du$$
$$= \frac{1}{3}\frac{u^{3/2}}{3/2} + C = \frac{2}{9}(x^3+1)^{3/2} + C$$

5. Evaluate: $\int \frac{\cos(x)}{1+4\sin(x)}\,dx$

$u = 1 + 4\sin x \Rightarrow du = 4\cos x\,dx$
$\Rightarrow \frac{du}{4} = \cos x\,dx$

$$= \frac{1}{4}\int \frac{du}{u}$$
$$= \frac{1}{4}\ln|u| + C = \frac{1}{4}\ln|1+4\sin x| + C$$