Name:
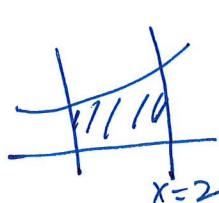
PS ID:

1. (2 pts) Let R be a region totally contained in the first quadrant with area 1/2 and centroid $C(14,10)$. If this region is rotated about the line $y = -1$, find the volume of the solid formed.

$$V = 2\pi \bar{R} A \qquad A = 1/2 \qquad \bar{R} = 10 - (-1) = 11$$

$$= 2\pi (11) \cdot \frac{1}{2} = 11\pi$$

2. (2 pts) Let R be the region bounded by $f(x) = e^{3x}$ and the x-axis, $0 \le x \le 2$. If R is the base of a solid whose cross sections perpendicular to the x-axis are squares, find the volume of this solid.

$$V = \int_0^2 (e^{3x})^2 dx = \int_0^2 e^{6x} dx = \frac{e^{6x}}{6} \Big|_0^2$$

$$= \frac{1}{6}(e^{12} - e^0) = \frac{1}{6}(e^{12} - 1)$$

$x = 2$

3. (1 pt) Let R be the region bounded by $x^2 + y^2 = 25$. If R is the base of a solid whose cross sections perpendicular to the y-axis are squares, **set up** an integral that gives the volume of this solid.

$$\text{side} = 2\sqrt{25 - y^2}$$

$$\text{Area} = 4(25 - y^2)$$

$$\text{volume} = \int_{-5}^{5} 4(25 - y^2) dy$$

4. (3pts) Find the particular solution: $y' = 24yx^3$, $y(0) = 3$. (Suppose y>0)

$$\frac{dy}{dx} = 24yx^3$$

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$$\ln|y| = 6x^4 + C$$

$$y = e^{6x^4 + C} = Ce^{6x^4}$$

$$3 = Ce^0 \Rightarrow C = 3$$

$$\therefore y = 3e^{6x^4}$$

5. (2pts) A bacteria population that grows exponentially doubles every 10 hours. If the initial population is 4,000 bacteria, find the population after 2 hours.

$$P = P_0 e^{rt}$$

$$2P_0 = P_0 e^{10r}$$

$$\ln(2) = 10r$$

$$r = \frac{1}{10}\ln(2)$$

$$P = 4000 e^{\frac{1}{10}\ln(2) \cdot 2}$$

$$= 4000 e^{\frac{1}{5}\ln(2)}$$

$$= 4000 e^{\ln(2)^{1/5}}$$

$$= 4000 (2)^{1/5}$$