Name:

Math 1432
Quiz 5 Version A

Ps ID:

(Each question is worth 2pts)

Evaluate the following integrals. Show your work clearly and box your final answer.

1. $\int x\ln(5x)\,dx =$

$u = \ln(5x) \qquad dv = x\,dx$

$du = \frac{1}{5x}\cdot 5\,dx \qquad v = \frac{x^2}{2}$

$= \frac{x^2}{2}\ln(5x) - \int \frac{x^2}{2}\frac{dx}{x} = \frac{x^2}{2}\ln(5x) - \frac{1}{2}\int x\,dx = \frac{x^2}{2}\ln(5x) - \frac{x^2}{4} + C$

2. $\int \arctan(x)\,dx =$

$u = \tan^{-1}x \qquad dv = dx$

$du = \frac{1}{1+x^2}dx \qquad v = x$

$\longrightarrow x\tan^{-1}x - \int \frac{x}{1+x^2}dx$

$= x\tan^{-1}x - \int \frac{1}{2u}du \qquad u = 1+x^2,\ du = 2x\,dx$

$= x\tan^{-1}x - \frac{1}{2}\ln|u| + C$

$= x\tan^{-1}x - \frac{1}{2}\ln|1+x^2| + C$

3. $\int \sec^4 x \tan^2 x\,dx =$

$= \int \sec^2 x \sec^2 x \tan^2 x\,dx$

$= \int \sec^2 x (1+\tan^2 x)\tan^2 x\,dx \qquad u = \tan x,\ du = \sec^2 x\,dx$

$= \int (1+u^2)u^2\,du = \int (u^2+u^4)\,du = \frac{u^3}{3} + \frac{u^5}{5} + C$

$= \frac{\tan^3 x}{3} + \frac{\tan^5 x}{5} + C$

4. $\int \sin^3 x \cos^2 x\,dx =$

$\int \sin x \sin^2 x \cos^2 x\,dx$

$= \int \sin x (1-\cos^2 x)\cos^2 x\,dx \qquad u = \cos x,\ du = -\sin x\,dx$

$= -\int (1-u^2)u^2\,du$

$= -\int u^2 - u^4\,du = -\frac{u^3}{3} + \frac{u^5}{5} + C = -\frac{\cos^3 x}{3} + \frac{\cos^5 x}{5} + C$

5. Use the appropriate trigonometric substitution to express the following integral in terms of θ (that is, switch the variable and stop, do not compute the integral).

$\int \frac{2}{x^2\sqrt{16-x^2}}dx =$

$x = 4\sin\theta$

$dx = 4\cos\theta\,d\theta$

$\int \frac{2\cdot 4\cos\theta\,d\theta}{16\sin^2\theta\sqrt{16-16\sin^2\theta}}$

$= \int \frac{8\cos\theta\,d\theta}{16\sin^2\theta\, 4\cos\theta} = \frac{1}{8}\int \csc^2\theta\,d\theta$

Name:

Math 1432
Quiz 5 Version B

Ps ID:
(Each question is worth 2pts)
Evaluate the following integrals. Show your work clearly and box your final answer.

1. $\int x\ln(6x)\,dx =$

See version A

2. $\int \arctan(x)\,dx =$

See version A

3. $\int \sec^4 x \tan^2 x\,dx =$

See version A

4. $\int \cos^3 x \sin^2 x\,dx =$ $\int \cos^2 x \cos x \sin^2 x\,dx$

$= \int (1-\sin^2 x)\sin^2 x \cos x\,dx$

$= \int (1-u^2)u^2\,du$

$= \int (u^2 - u^4)\,du$

$= \frac{u^3}{3} - \frac{u^5}{5} + C = \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + C$

$u = \sin x$
$du = \cos x\,dx$

5. Use the appropriate trigonometric substitution to express the following integral in terms of θ (that is, switch the variable and stop, do not compute the integral).

$\int \frac{4}{x^2\sqrt{9+x^2}}\,dx =$ $4\int \frac{3\sec^2\theta\,d\theta}{9\tan^2\theta\sqrt{9+9\tan^2\theta}}$

$= \frac{4}{9}\int \frac{3\sec^2\theta\,d\theta}{\tan^2\theta\, 3\sec\theta}$

$= \frac{4}{9}\int \frac{\sec\theta}{\tan^2\theta}\,d\theta = \frac{4}{9}\int \cot^2\theta \sec\theta\,d\theta$

$x = 3\tan\theta$
$dx = 3\sec^2\theta\,d\theta$