Lab 12/22

(*) $\displaystyle\int \frac{\arcsin(5x)}{\sqrt{1-25x^2}}\,dx$

$u = \sin^{-1} 5x$

$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$

$\frac{du}{dx} = \frac{5}{\sqrt{1-25x^2}}$

$\frac{du}{5} = \frac{dx}{\sqrt{1-25x^2}}$

$= \frac{1}{5}\int u\,du$

$= \frac{u^2}{10} + C$

$= \frac{(\sin^{-1} 5x)^2}{10} + C$

(*) $\displaystyle\int \frac{e^{3x}}{1+e^{6x}}\,dx$

$u = e^{3x}$

$du = 3e^{3x}\,dx$

$\frac{du}{3} = e^{3x}\,dx$

$= \displaystyle\int \frac{e^{3x}}{1+(e^{3x})^2}\,dx$

$= \frac{1}{3}\displaystyle\int \frac{du}{1+u^2}$

$= \frac{1}{3}\tan^{-1}u + C$

$= \frac{1}{3}\tan^{-1}(e^{3x}) + C$

$\displaystyle\int_0^{\ln(6)} \frac{e^{3x}}{1+e^{6x}}\,dx$

$= \frac{1}{3}\tan^{-1}(e^{3x})\Big|_0^{\ln 6}$

$= \frac{1}{3}\left[\tan^{-1}(e^{3\ln 6}) - \tan^{-1}(e^0)\right]$

$= \frac{1}{3}\left[\tan^{-1}(6^3) - \frac{\pi}{4}\right]$

(x) $f(x) = -20 + 9x - x^2$ $\qquad$ x-axis

$y = -20 + 9x - x^2$ $\qquad$ $y = 0$

$-20 + 9x - x^2 = 0$

$x^2 - 9x + 20 = 0$ $\qquad$ 20 $\qquad$ 1×20, 2×10, 4×5.

$x^2 - 5x - 4x + 20 = 0$

$x(x-5) - 4(x-5) = 0$

$(x-5)(x-4) = 0$

$x = 5$ or 4

$$\int_4^5 -20 + 9x - x^2 \, dx$$

$ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$x^2 - 9x + 20 = 0$

$b = -9 \quad a = 1 \quad c = 20$

$$\int_0^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}} \qquad \int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a} + C$$

$$= \sin^{-1}\frac{x}{2}\Big|_0^{\sqrt{3}}$$

$$= \sin^{-1}\frac{\sqrt{3}}{2}$$

$$= \frac{\pi}{3}$$

(*) $f(x) = |8x| \quad [-2, 2]$

$$f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$Av = \frac{1}{2-(-2)} \int_{-2}^{2} |8x|\,dx$$

$$= \frac{1}{4} \int_{-2}^{2} |8x|\,dx$$

$$= \frac{1}{4}\left[\int_{-2}^{0} -8x + \int_0^2 8x\,dx\right] = \frac{1}{4}\left[-4x^2\Big|_{-2}^{0} + 4x^2\Big|_0^2\right]$$

$$= \frac{1}{4}[16 + 16] = \frac{32}{4} = 8$$

$|8c| = 8$

$-8c = 8 \qquad 8c = 8$

$c = -1 \qquad c = 1$

(*) Avg value of $f(x) = 12$

$f(x)$ is even $[-2, 2]$

$\int_{-2}^{0} f(x)\,dx$?

$$\text{Avg value} = \frac{1}{4}\int_{-2}^{2} f(x)\,dx$$

$$12 = \frac{1}{4}\left[\int_{-2}^{0} f(x)\,dx + \int_{0}^{2} f(x)\,dx\right]$$

$$48 = 2\int_{-2}^{0} f(x)\,dx$$

$$24 = \int_{-2}^{0} f(x)\,dx$$

(*) $$\lim_{x\to 2} \frac{x-2}{x^2-4}$$

$$= \lim_{x\to 2} \frac{\cancel{(x-2)}}{\cancel{(x-2)}(x+2)}$$

$$= \lim_{x\to 2} \frac{1}{x+2}$$

$$= \frac{1}{4}$$

(*) $\lim_{x\to 0} \frac{3x}{\sin 2x}$

$\lim_{x\to 0} \frac{\sin ax}{bx} = \frac{a}{b}$

$= \lim_{x\to 0} \frac{1}{\frac{\sin 2x}{3x}}$

$= \frac{1}{\frac{2}{3}}$

$= \frac{3}{2}$

(*) $f(x) = x^3 + 2x - 1$

$(f^{-1})'(b) = \frac{1}{f'(a)}$

$(f^{-1})'(2)$?

$f(a) = b$

$a^3 + 2a - 1 = 2$

$a = 1$

$f'(x) = 3x^2 + 2$

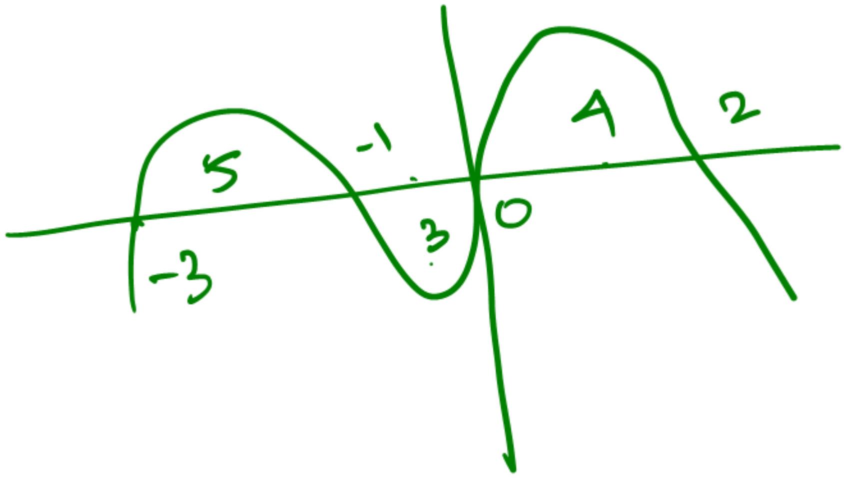
$f'(1) = 3 + 2 = 5$

$(f^{-1})'(2) = \frac{1}{5}$

(*) $F(x) = 4x + \int_0^{5x} \frac{1}{1+t^2}\,dt$

$F'(x)$ $F'(x) = 4 + \frac{1}{1+(5x)^2}\frac{d}{dx}(5x) - \frac{1}{1+(0)}\frac{d(0)}{dx}$

$= 4 + \frac{5}{1+25x^2}$



Area $[-3,-1] = 5$

" $[-1, 0] = 3$

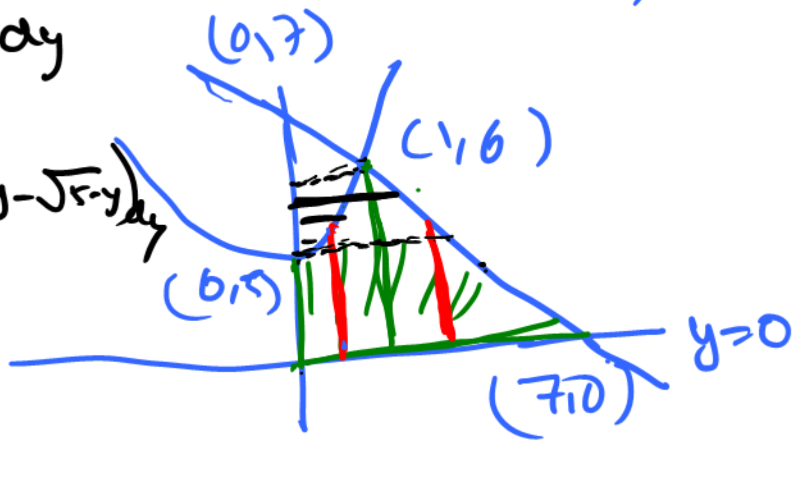
" $[0, 2] = 4$

$$\int_{-3}^{2} f(x)\,dx = 5-3+4 = 6$$

(*) $f(x) = \begin{cases} x^2+5, & 0 \le x \le 1 \\ 7-x, & 1 < x \le 7 \end{cases}$

$\int_0^5 7-y\,dy$

$+\int_5^6 (7-y-\sqrt{5-y})\,dy$



$y = x^2+5$

$x+y = 7$

$\int_0^1 x^2+5\,dx + \int_1^7 7-x\,dx$

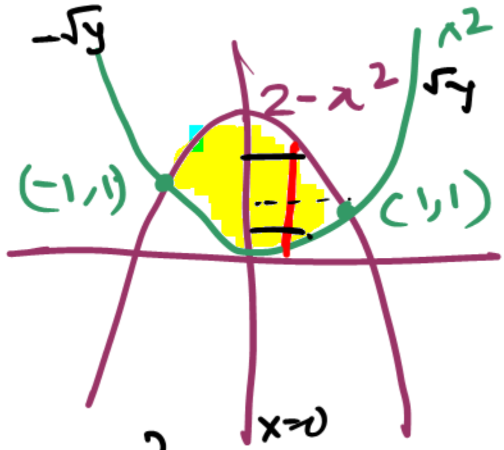
$x^2+5 = 7-x$

$x^2+x-2 = 0$

$x^2+\cancel{2x}-x-2 = 0$

$(x+2)(x-1) = 0 \quad x = -2, 1$

(*) $y = 2 - x^2 \qquad y = x^2$

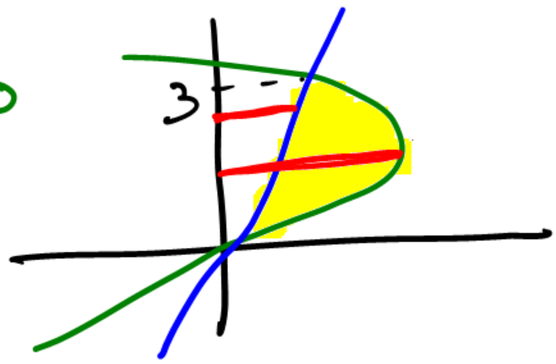


$$2 - x^2 = x^2$$
$$2 = 2x^2$$
$$1 = x^2$$
$$\pm 1 = x$$

$$2\left[\int_0^1 \sqrt{y}\,dy + \int_1^2 \sqrt{2-y}\,dy\right]$$

$$2\int_0^1 2 - x^2 - x^2\,dx$$

(*) $4x = 4y - y^2 \qquad 4x - y = 0$



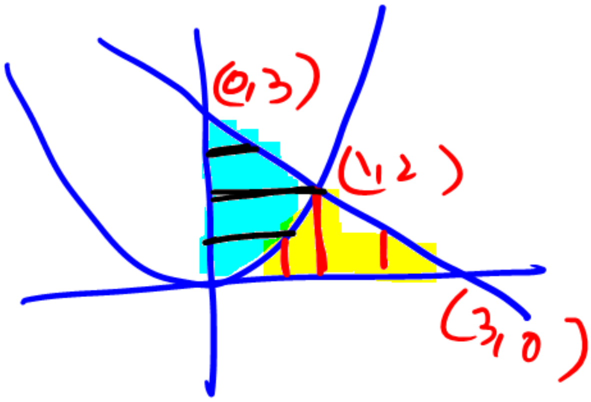
$$\int_0^3 \frac{4y - y^2}{4} - \frac{y}{4}\,dy$$

$$\frac{4y - y^2}{\cancel{4}} = \frac{y}{\cancel{4}}$$

$$y^2 + y - 4y = 0$$

$$y^2 - 3y = 0 \qquad y(y-3) = 0 \qquad y = 0, 3$$

⊛ $y = 2x^2$ $x + y = 3$



x-axis

$$\int_0^1 2x^2\,dx + \int_1^3 3 - x\,dx$$

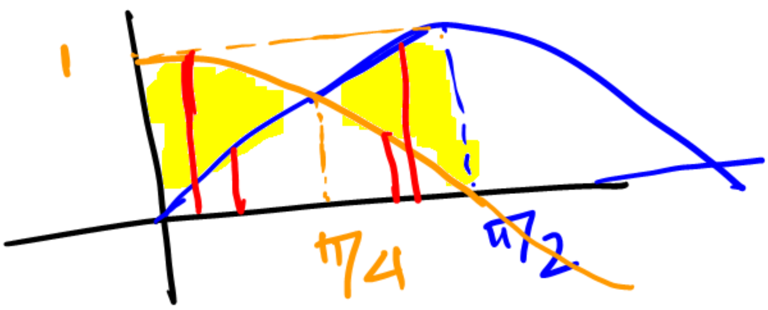
y-axis

$$\int_0^2 \sqrt{\frac{y}{2}}\,dy + \int_2^3 3 - y\,dy$$

⊛ $f(x) = \sin x$ $g(x) = \cos x$ $[0, \pi/2]$

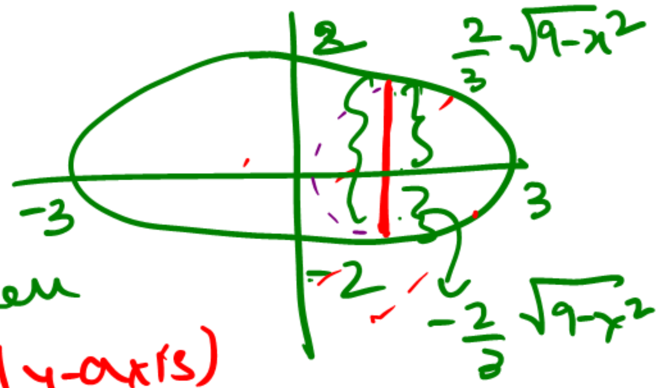
$$\int_0^{\pi/4} \cos x - \sin x\,dx$$

$$+ \int_{\pi/4}^{\pi/2} \sin x - \cos x\,dx$$



	0	30	45	60	90
sin	$\sqrt{\frac{0}{4}}$	$\sqrt{\frac{1}{4}}$	$\sqrt{\frac{2}{4}}$	$\sqrt{\frac{3}{4}}$	$\sqrt{\frac{4}{4}}$
	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undef

(*) $\frac{x^2}{9} + \frac{y^2}{4} = 1$



Volume formed

cross sec $\perp$ x-ax when

(|| x-axis ⇒ ⊥ y-axis)

(i) square.

side $= \frac{4}{3}\sqrt{9-x^2}$

$$A_s = (\text{side})^2$$

$$= \left(\frac{4}{3}\sqrt{9-x^2}\right)^2$$

$$= \frac{16}{9}(9-x^2)$$

$$V = 2\int_0^3 \frac{16}{9}(9-x^2)\,dx$$

(ii) diameter of semi circle

Diameter $= \frac{4}{3}\sqrt{9-x^2}$

$r = \frac{2}{3}\sqrt{9-x^2}$

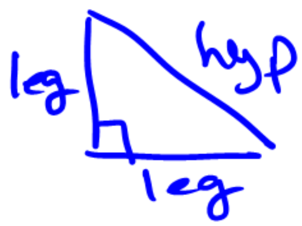
$$A_{sc} = \frac{\pi r^2}{2}$$

$$= \frac{\pi}{2}\left(\frac{2}{3}\sqrt{9-x^2}\right)^2$$

$$= \frac{2\pi}{9}(9-x^2)$$

$$V = 2\int_0^3 \frac{2\pi}{9}(9-x^2)\,dx$$

(*) one of the leg rt ∠ed Δ



leg $= \frac{4}{3}\left(\sqrt{9-x^2}\right)$

$$A_{r.\Delta} = \frac{1}{2}(\text{leg})^2$$

$$= \frac{1}{2}\left(\frac{4}{3}\sqrt{9-x^2}\right)^2$$

$$= \frac{8}{9}(9-x^2)$$

$$V = 2\int_0^3 \frac{8}{9}(9-x^2)\,dx$$