Lab 12/23

* $f(x) = 3x^3 \quad [-1,1]$

$$Av = \frac{1}{2}\int_{-1}^{1} 3x^3\,dx$$

$$= \frac{1}{2}\,\frac{3}{4}\,x^4\Big|_{-1}^{1}$$

$$= \frac{3}{8}\left[1^4 - (-1)^4\right]$$

$$= \frac{3}{8}\left[1-1\right] = 0$$

$$3x^3 = 0 \Rightarrow x = 0$$

* $3\sin x$

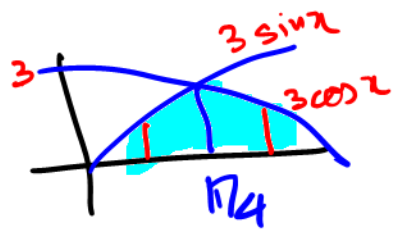
$3\cos x$

Area between the curves

$$\int_0^{\pi/4} 3\cos x - 3\sin x\,dx$$

$$+ \int_{\pi/4}^{\pi/3} 3\sin x - 3\cos x\,dx$$

Area between curves & x-axis



$$\int_0^{\pi/4} 3\sin x\,dx + \int_{\pi/4}^{\pi/3} \cos x\,dx$$

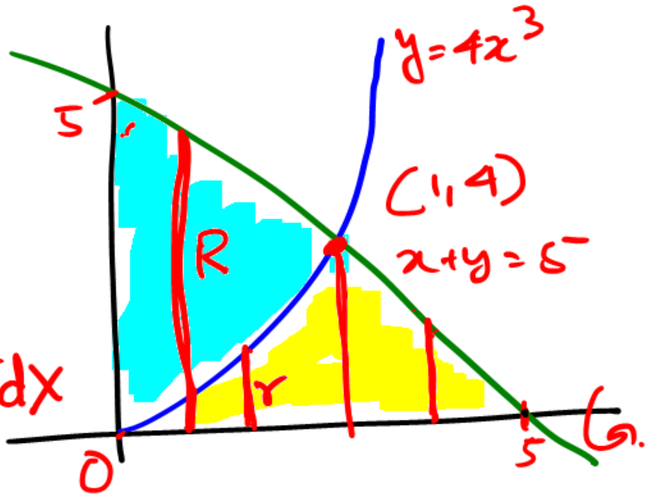
$$= 3\left[-\cos x\Big|_0^{\pi/4} + \sin x\Big|_{\pi/4}^{\pi/3}\right]$$

$$= 3\left[-\frac{\sqrt{2}}{2} + 1 + \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\right] = 3 + \frac{3\sqrt{3}}{2} - 3\sqrt{2}$$

Volume

9. $y = 4x^3 \quad x + y = 5$

x-axis ⟲.



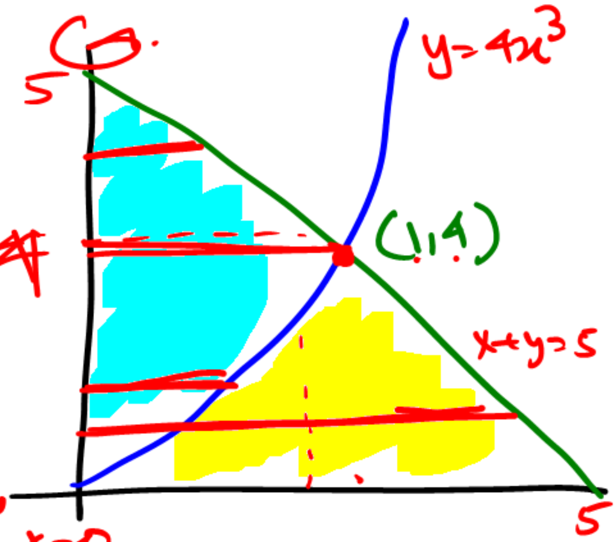
y=0 (D) $\rightarrow \pi \int_0^1 (4x^3)^2 dx + \pi \int_1^5 (5-x)^2 dx$

x	0	5
y	5	0

$5 - x = 4x^3$

x=0 (W) $\pi \int_0^1 (5-x)^2 - (4x^3)^2 dx$

y-axis ⟲.

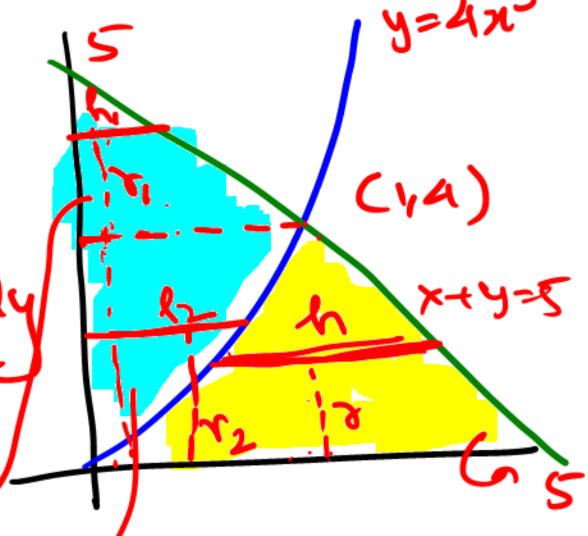


x=0 (W) $\pi \int_0^4 (5-y)^2 - \left(\frac{y}{4}\right)^{2/3} dy$ ✱

y=0 (D) $\pi \int_0^4 \left(\frac{y}{4}\right)^{2/3} dy + \pi \int_4^5 (5-y)^2 dy$ x=0

x-axis ⟲

$\int 2\pi r h$



y=0 Shell

$2\pi \int_0^4 y\left(5 - y - \left(\frac{y}{4}\right)^{1/3}\right) dy$

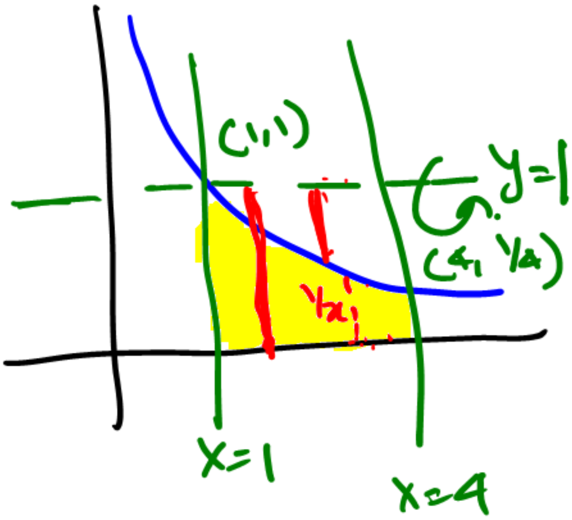
x=0 $2\pi \int_4^5 (4+y)(5-y) dy$

$+ \; 2\pi \int_0^4 y \left(\frac{y}{4}\right)^{1/3} dy$

49 $y = \frac{1}{x}$ $y=0$ $x=1$ $x=4$

$y=1$ ca.

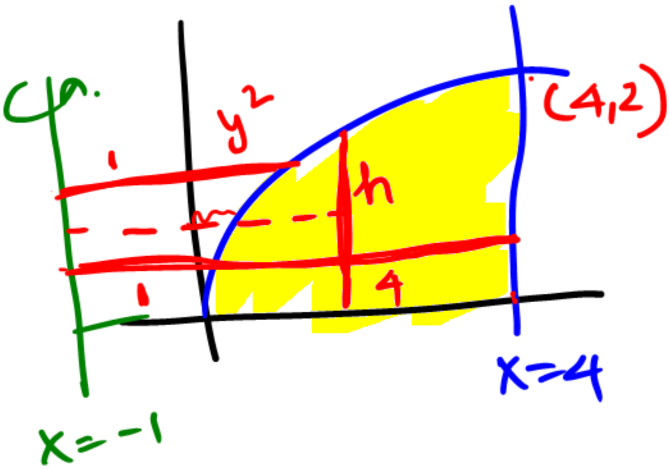
$$\pi \int_1^4 1^2 - \left(1 - \frac{1}{x}\right)^2 dx$$



51 $y = \sqrt{x}$ $y=0$ $x=0$ $x=4$

$x=-1$ ca.

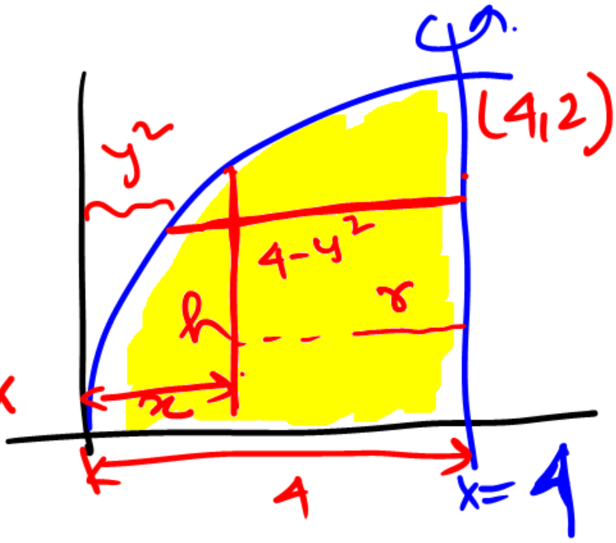
D/W $\pi \int_0^2 (5)^2 - (1+y^2)^2 \, dy$



$x=4$ ca.

D/W $\pi \int_0^2 (4-y^2)^2 \, dy$

Shell $2\pi \int_0^4 (4-x)(\sqrt{x}) \, dx$



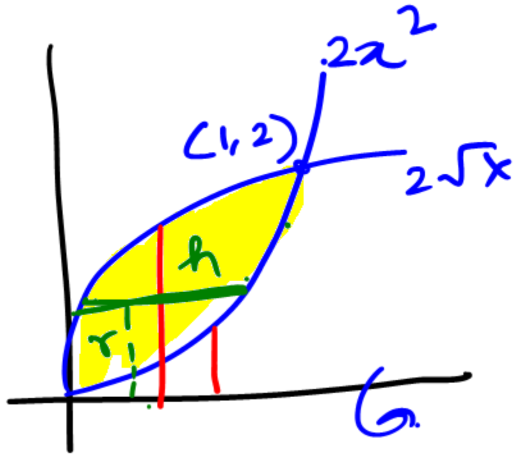
(*) $SA = \int 2\pi f(x) \sqrt{1+(f'(x))^2} \, dx$

$= \int_0^2 2\pi \sqrt{x} \sqrt{1 + \frac{1}{4x}} \, dx$

$f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2\sqrt{x}}$

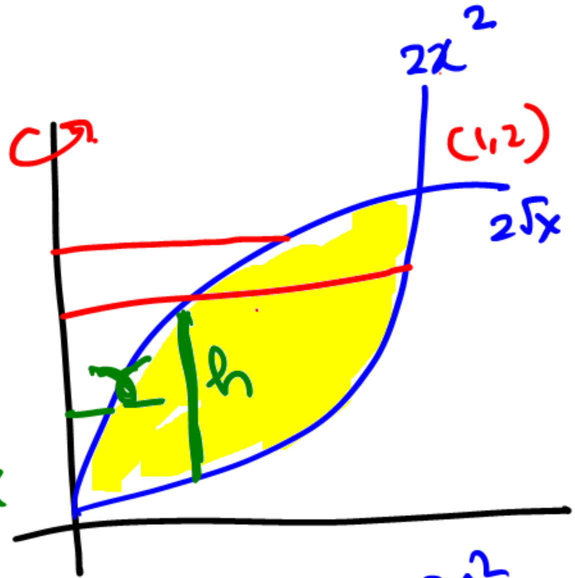
$y = 2x^2$ $\quad y = 2\sqrt{x}$



x-axis $\circlearrowleft$

D/W $\pi\int_0^1 (2\sqrt{x})^2 - (2x^2)^2\,dx$

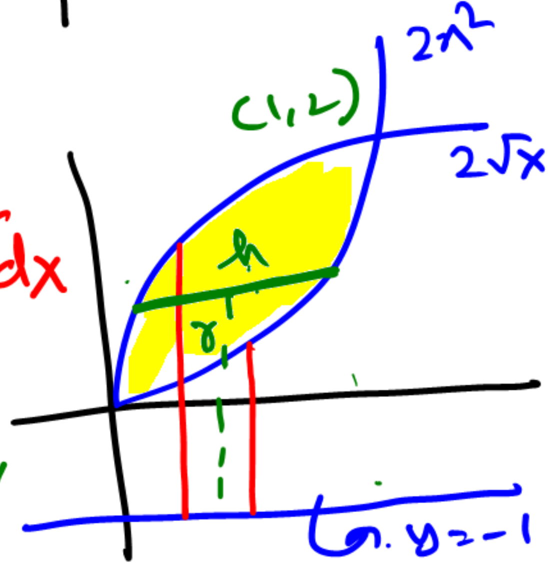
shell $2\pi\int_0^2 \underbrace{y}_{r}\Big(\underbrace{\sqrt{\tfrac{y}{2}} - \tfrac{y^2}{4}}_{h}\Big)dy$



y-axis $\circlearrowleft$

D/W $\pi\int_0^2 \left(\sqrt{\tfrac{y}{2}}\right)^2 - \left(\tfrac{y^2}{4}\right)^2 dy$

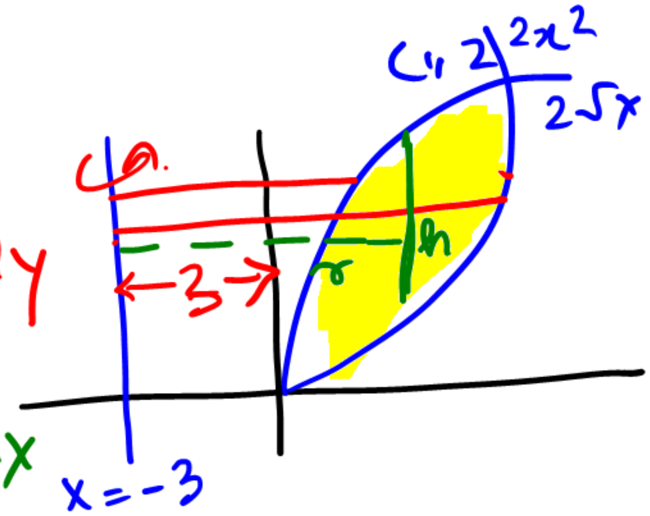
Shell $2\pi\int_0^1 x\,(2\sqrt{x} - 2x^2)\,dx$



$y = -1$ $\circlearrowleft$

D/W $\pi\int_0^1 (1+2\sqrt{x})^2 - (1+2x^2)^2\,dx$

shell $2\pi\int_0^2 (1+y)\left(\sqrt{\tfrac{y}{2}} - \tfrac{y^2}{4}\right)dy$

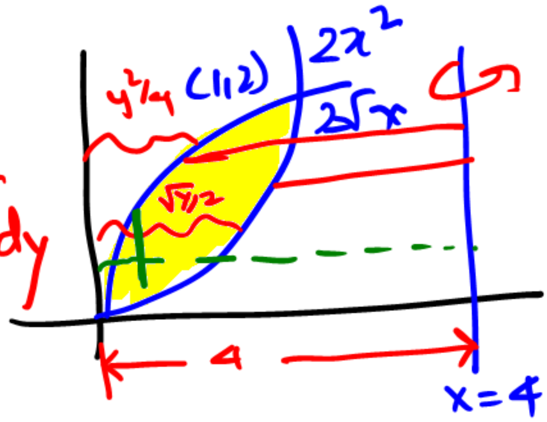


$x = -3$ $\circlearrowleft$

D/W $\pi\int_0^2 \left(3+\sqrt{\tfrac{y}{2}}\right)^2 - \left(3+\tfrac{y^2}{4}\right)^2 dy$

shell $2\pi\int_0^1 (3+x)(2\sqrt{x} - 2x^2)\,dx$

$y = 2x^2 \qquad y = 2\sqrt{x}$



x = 4 (a)

D/W $\pi\int_0^2 \left(4-\frac{y^2}{4}\right)^2 - \left(4-\sqrt{\frac{y}{2}}\right)^2 dy$

Shell

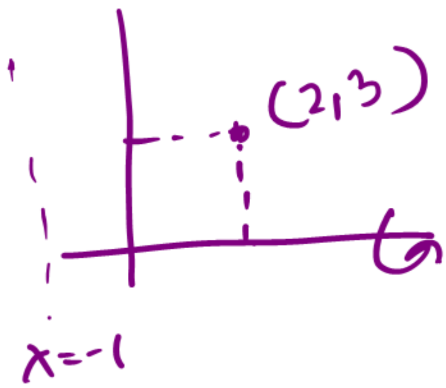
$2\pi\int_0^1 (4-x)(2\sqrt{x}-2x^2)\,dx$

$y = 2$ (a) ← Try this

Pappu's Thm

$V = 2\pi A R$

suppose centroid (2, 3) $\quad A = 10$



V rotating x-axis

$V = 2.\pi.10.3$

$= 60\pi$

y axis (a) $V = 2\pi.10.2$

$= 40\pi$

x = -1 (a) $V = 2\pi.10.(2+1)$

$= 60\pi$

(*) $\ln x \dfrac{dy}{dx} = \dfrac{y}{x}$

$$\int \frac{dy}{y} = \int \frac{dx}{x \ln x}$$

$u = \ln x$

$du = \frac{1}{x} dx$

$$\ln|y| = \int \frac{du}{u}$$

$$\ln|y| = \ln|u| + c$$

$$y = e^{\ln|u| + c}$$

$$= e^{\ln|u|} \cdot e^{c}$$

$$y = Cu = c\ln x$$

$y(e) = 2$

$$2 = c \ln e$$

$$2 = c$$

$$y = 2\ln x$$

D : $\int_a^b \pi R^2 \, dx$

W $= \pi \int_a^b R^2 - r^2 \, dx$

Shell $2\pi \int_a^b r(x) h(x) \, dx$

(*) $\frac{dy}{dx} = e^{y-x}$

$\frac{dy}{dx} = e^{y} \cdot e^{-x}$

$\int e^{-y} dy = \int e^{-x} dx$

$\frac{e^{-y}}{-1} = \frac{e^{-x}}{-1} + C$

$e^{-y} = e^{-x} + C$

$\ln(e^{-y}) = \ln(e^{-x} + C)$

$-y = \ln|e^{-x} + C|$

$y = -\ln|e^{-x} + C|$

(*)

$$\frac{dy}{dx} = \frac{y^2+1}{xy+y}$$

$$\frac{dy}{dx} = \frac{y^2+1}{y(x+1)}$$

$$\int \frac{y}{y^2+1}\,dy = \int \frac{dx}{x+1}$$

$u = y^2+1$
$du = 2y\,dy$

$$\frac{1}{2}\int \frac{du}{u} = \ln|x+1| + C$$

$$\frac{1}{2}\ln|u| = \ln|x+1| + C$$

$$\frac{1}{2}\ln|y^2+1| = \ln|x+1| + C$$

$$\ln|y^2+1| = 2\ln|x+1| + C$$

$$y^2+1 = e^{2\ln|x+1|+C}$$

$$y^2 = C(x+1)^2 - 1$$

$$y = \sqrt{C(x+1)^2-1}$$

(*) $f(x) = 12x - 6x^2 \quad [0, 5]$

$$AV = \frac{1}{5}\int_0^5 12x - 6x^2$$

$$= \frac{1}{5}\left(6x^2 - 2x^3\right)\Big|_0^5$$

$$= \frac{1}{5}(150 - 250)$$

$$= -\frac{100}{5} = -20$$

$$12x - 6x^2 = -20$$

$$6x^2 - 12x - 20 = 0$$

$$3x^2 - 6x - 10 = 0$$

$$x = \frac{6 \pm \sqrt{36 + 120}}{6}$$

$$= \frac{6 \pm \sqrt{156}}{6}$$

$$= \frac{6 \pm \sqrt{3 \cdot 4 \cdot 13}}{6} = \frac{6 \pm 2\sqrt{39}}{6}$$

$$= \frac{3 \pm \sqrt{39}}{3}$$

6 | 156
2 | 26
13