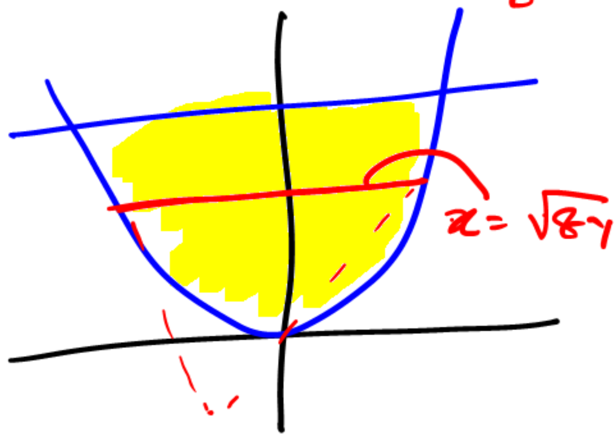
Lab 12/24

(*)



Side $= 2\sqrt{8y}$

$= 4\sqrt{2y}$

Area of Equi Δ

$= \frac{\sqrt{3}}{4} a^2$

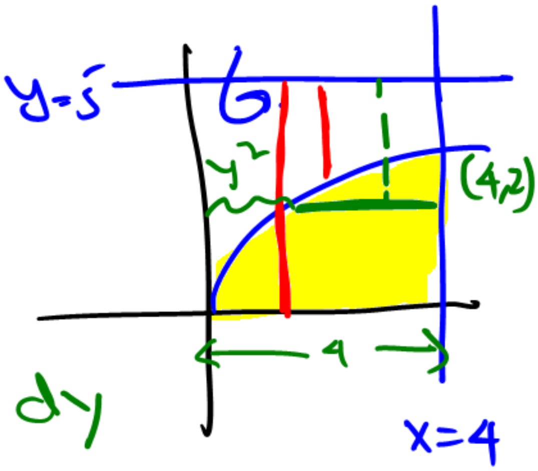
$= \frac{\sqrt{3}}{4}\left(4\sqrt{2y}\right)^2 = \frac{\sqrt{3}}{4} 16 \cdot 2y$

$= 8\sqrt{3}y$

$$V = \int_0^4 8\sqrt{3}y\,dy$$

(*) $f(x) = \sqrt{x}$, $\cancel{y} = 0$ $x \in [0, 4$

$y = 5$ ↺.

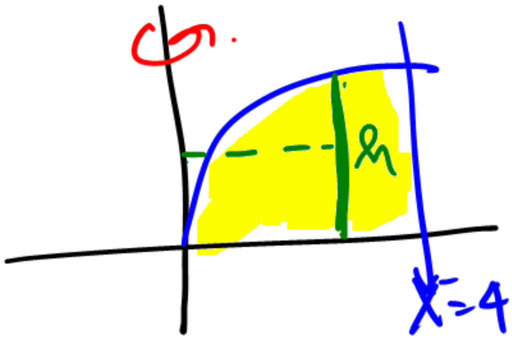


D/W $\pi \int_0^4 5^2 - (5-\sqrt{x})^2\,dx$

Shell $2\pi \int_0^2 (5-y)(4-y^2)\,dy$

(*) $y = 4\sqrt{x}$ $x=4$ $y=0$

y-axis ↻.



Shell

$$2\pi \int_0^4 x \, 4\sqrt{x} \, dx$$

$$= 2\pi \int_0^4 4x^{3/2} \, dx = 8\pi \int_0^4 x^{3/2} dx$$

(*) curve length $f(x) = \ln(4\sec x)$ $0 \le x \le \pi/3$

$$\int_0^{\pi/3} \sqrt{1 + (f'(x))^2} \, dx$$

$f(x) = \ln(4\sec x)$

$$f'(x) = \frac{4\sec x \tan x}{4\sec x}$$

$f'(x) = \tan x$

$$= \int_0^{\pi/3} \sqrt{1 + \tan^2 x} \, dx$$

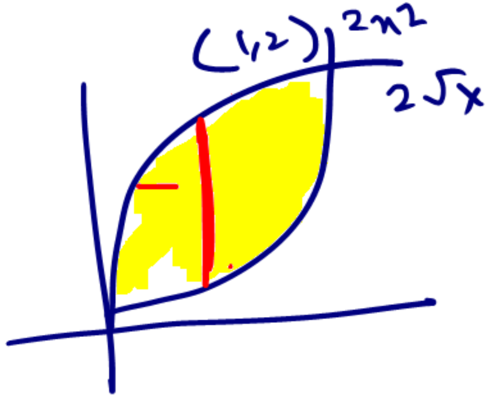
$$= \int_0^{\pi/3} \sqrt{\sec^2 x} \, dx$$

$$= \int_0^{\pi/3} \sec x \, dx = \ln|\sec x + \tan x| \Big|_0^{\pi/3}$$

$$= \ln|2 + \sqrt{3}| - \ln|1 + 0|$$

$$= \ln|\sqrt{3} + 2|$$

⊛ $y=2x^2$ $\quad y=2\sqrt{x}$



squares ⊥ x-axis

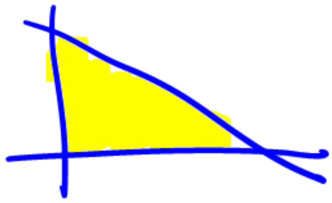
side $= 2\sqrt{x} - 2x^2$

$$A = \left(2\sqrt{x} - 2x^2\right)^2$$

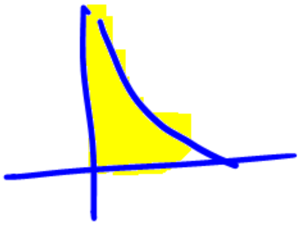
$$V = \int_0^1 \left(2\sqrt{x} - 2x^2\right)^2 dx$$

⊛ $\displaystyle\int_1^3 \frac{1}{x-2}\,dx$ $\quad$ inf dis $x=2$

$$= \int_1^2 \frac{1}{x-2}\,dx + \int_2^3 \frac{1}{x-2}\,dx$$



$$= \lim_{b\to 2^-} \int_1^b \frac{1}{x-2}\,dx + \lim_{a\to 2^+} \int_a^3 \frac{1}{x-2}\,dx$$



$$= \lim_{b\to 2^-} \ln|x-2| \Big|_1^b + \lim_{a\to 2^+} \ln|x-2| \Big|_a^3$$

$$= \underbrace{\lim_{b\to 2^-} \ln|b-2|} - \ln|1-2| + \ln|3-2| \underbrace{- \lim_{a\to 2^+} |a-2|}$$

$\ln(1) = 0 = \ln(1)$

Diverges

⊗

$A = A_0 e^{kt}$

A ↑ Value at time t

A_0 → initial i.e value at $t=0$

2 conditions
using one rate? → Ques is asking

$A = 2A_0$

$2\cancel{A_0} = \cancel{A_0} e^{kt}$

$2 = e^{kt}$ $\quad k = 10\%$

$\ln(2) = kt$

$t = \frac{\ln(2)}{k} = \frac{\ln(2)}{0.1} = 10\ln(2)$

⊛ 22

Half life = 1620 years

current = 100 gr $\quad$ mass? 200 years ago.

$A = A_0 e^{kt}$

$\frac{\cancel{A_0}}{2} = \cancel{A_0} e^{kt}$

$\frac{1}{2} = e^{kt}$

$k = \frac{\ln(1/2)}{t} = \frac{\ln(1/2)}{1620}$

$$k = \frac{\ln(1/2)}{1620}$$

current 100 gr   mass? 200 years ago

$$A = A_0 e^{kt}$$

$$100 = A_0 e^{\frac{\ln(1/2)}{1620} 200}$$

$$\frac{200 \ln(1/2)}{1620}$$

$$100 = A_0 \left(\frac{1}{2}\right)^{\frac{200}{1620}}$$

$$\ln\left(\frac{1}{2}\right)^{\frac{200}{1620}}$$

$$A_0 = 100 (2)^{\frac{200}{1620}}$$

⊛ b How long? reduce by 75%

$$A = A_0 e^{kt}$$

$$A = 100\%$$

$$= A_0 e^{\frac{\ln(1/2)}{1620} t}$$

$$A_0 = 25\%$$

$$A = A_0 \left(\frac{1}{2}\right)^{\frac{t}{1620}}$$

$$25 = 100 \left(\frac{1}{2}\right)^{t/1620}$$

$$\frac{1}{4} = \left(\frac{1}{2}\right)^{t/1620}$$

$$\ln\left(\frac{1}{4}\right) = \frac{t}{1620} \ln\left(\frac{1}{2}\right)$$

$$t = \frac{1620 \ln(1/4)}{\ln(1/2)}$$

30 ⊗

$$P = P_0 e^{kt}$$

$P_0 = 10{,}000$

1 week $\rightarrow 12{,}000$

$$12000 = 1000\, e^{k \cdot 1}$$

$$1.2 = e^{k}$$

$$k = \ln(1.2)$$

$$P = P_0 e^{\ln(1.2) \cdot t}$$

$$P(t) = P_0 (1.2)^t = 10000 (1.2)^t$$

(b) How many after one year?

$$P(52) = 1000 (1.2)^{52}$$

⊛ 34

$A_0 = 100$

$10\% \rightarrow$ 5 mins

$$A = A_0 e^{kt}$$

$$90 = 100\, e^{k \cdot 5}$$

$$0.9 = e^{5k} \qquad k = \frac{1}{5} \ln(0.9)$$

$$A = A_0 e^{kt} = A_0 e^{\frac{\ln(0.9)}{5} t}$$

$$A(t) = 100 (0.9)^{t/5}$$

$t = 20$

$$A(20) = 100 (0.9)^{20/5}$$

$$= 100 (0.9)^4$$

Improper Intg

3. $\displaystyle\int_1^9 (x-1)^{-2/3}\,dx$ inf disc at $x=1$

$$= \lim_{a\to 1^-} \int_a^9 (x-1)^{-2/3}\,dx$$

$u = x-1$

$du = dx$

$\int (x-1)^{-2/3}\,dx$

$$= \int u^{-2/3}\,du = \frac{u^{1/3}}{1/3} + C = 3(x-1)^{1/3} + C$$

$$= \lim_{a\to 1^-} 3(x-1)^{1/3}\Big|_a^9$$

$$= 3(9-1)^{1/3} - \lim_{a\to 1^-} 3(a-1)^{1/3}$$

$$= 6$$

25 $$\int_e^{\infty} \frac{1}{x(\ln x)^2}\,dx$$

the upper limit is not bounded

$$= \lim_{b\to\infty} \int_e^{b} \frac{1}{x(\ln x)^2}\,dx$$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$$\int \frac{1}{x(\ln x)^2}\,dx$$

$$= \int \frac{du}{u^2} = -\frac{1}{u} + c = \frac{-1}{\ln x} + c$$

$$\lim_{b\to\infty} -\frac{1}{\ln x}\Big|_e^b = \lim_{b\to\infty} \frac{-1}{\ln b} + \frac{1}{\ln(e)} = 1$$

$$b\to\infty \Rightarrow \ln b \to \infty \Rightarrow \frac{1}{\ln b} \to 0$$

(*) 17 $\displaystyle\int_0^2 \frac{dx}{\sqrt{4-x^2}}$ inf disc at $x=2$

$$= \lim_{b\to 2^+} \int_0^b \frac{dx}{\sqrt{4-x^2}}$$

$$= \lim_{b\to 2^+} \sin^{-1}\left(\frac{x}{2}\right)\Big|_0^b$$

$$= \lim_{b\to 2^+} \sin^{-1}\left(\frac{b}{2}\right) - \sin^{-1}\left(\frac{0}{2}\right)$$

$$= \frac{\pi}{2}$$

(*) $\displaystyle\int_{-\infty}^{\infty} \frac{1}{1-x^2}\,dx$ $-\infty, \infty$

$x = \pm 1$
inf disc.

$$= \int_{-\infty}^{-2} + \int_{-2}^{-1} + \int_{-1}^{0} + \int_{0}^{1} + \int_1^2 + \int_2^{\infty}$$