Lab 12/29

$$\int u\,dv = uv - \int v\,du$$

(*) $\int_0^{\pi/2} 8x^3 \sin x\,dx$

$\int x^3 \sin x\,dx$ $\quad u = x^3 \quad dv = \sin x\,dx$

$du = 3x^2\,dx \quad v = -\cos x$

$= -x^3\cos x + 3\int x^2 \cos x\,dx$

$= -x^3\cos x + 3\left[x^2\sin x - 2\int x\sin x\,dx\right]$ $\quad u = x^2 \quad dv = \cos x\,dx$

$du = 2x\,dx \quad v = \sin x$

$= -x^3\cos x + 3x^2\sin x - 6\int x\sin x\,dx$

$= -x^3\cos x + 3x^2\sin x - 6\left[-x\cos x + \int \cos x\,dx\right]$ $\quad u = x$

$du = dx$

$dv = \sin x\,dx$

$= -x^3\cos x + 3x^2\sin x + 6x\cos x - 6\sin x + C$ $\quad v = -\cos x$

$$\int_0^{\pi/2} 8x^3\sin x\,dx = -8x^3\cos x + 24x^2\sin x + 48x\cos x - 48\sin x \Big|_0^{\pi/2}$$

(*) $\displaystyle\int \frac{x+8}{\sqrt{x^2+16x+11}}\,dx$

$u = x^2+16x+11$

$du = (2x+16)\,dx$

$du = 2(x+8)\,dx$

$x^2+16x+11$

$= x^2+2\cdot 8\cdot x+64+11-64$

$= (x+8)^2-53$

$= \frac{1}{2}\displaystyle\int \frac{du}{\sqrt{u}}$

$= \frac{1}{2}\displaystyle\int u^{-1/2}\,du$

$= \frac{1}{2}\,\frac{u^{1/2}}{1/2} + C = \sqrt{u} + C$

$= \sqrt{x^2+16x+11} + C$

(*) $\displaystyle\int \frac{2x^2}{(x^2+6)^{3/2}}\,dx$

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \csc^2 x$

$x = \sqrt{6}\tan\theta \qquad dx = \sqrt{6}\sec^2\theta\, d\theta$

$x^2 = 6\tan^2\theta$

$(6\tan^2\theta + 6)^{3/2}$

$= (6(\tan^2\theta+1))^{3/2}$

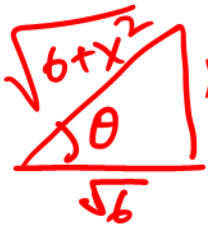
$= 6^{3/2}(\sec^2\theta)^{3/2}$

$= 6\cdot\sqrt{6}\sec^3\theta$

$\displaystyle\int \frac{2\cdot 6\tan^2\theta\,\sqrt{6}\sec^2\theta\, d\theta}{(6\tan^2\theta+6)^{3/2}}$

$= \displaystyle\int \frac{2\cdot\cancel{6}\cdot\cancel{\sqrt{6}}\,\tan^2\theta\,\cancel{\sec^2\theta}\, d\theta}{\cancel{6\sqrt{6}}\,\sec^{\cancel{3}}\theta} =$

$= 2\displaystyle\int \frac{\sin^2\theta}{\cos^2\theta\,\sec\theta}\, d\theta = 2\int \frac{\sin^2\theta}{\cos\theta}\, d\theta$



$= 2\displaystyle\int \frac{1-\cos^2\theta}{\cos\theta}\, d\theta$

$= 2\displaystyle\int \frac{1}{\cos\theta} - \frac{\cos^2\theta}{\cos\theta}\, d\theta$

$= 2\left[\int \sec\theta\, d\theta - \int\cos\theta\, d\theta\right]$

$= 2\ln\left|\frac{\sqrt{x^2+6}}{\sqrt{6}} + \frac{x}{\sqrt{6}}\right| - 2\frac{x}{\sqrt{x^2+6}} + C$

$= 2\ln|\sec\theta+\tan\theta| - 2\sin\theta + C = 2\ln\left|\frac{\sqrt{x^2+6}+x}{\sqrt{6}}\right| - \frac{2x}{\sqrt{x^2+6}} + C$

⊗ $A_0 = 3000$ $\quad A_0 \to 2A_0$ 60 mins
1 hr.

$$A = A_0 e^{kt}$$
$$2A_0 = A_0 = e^{1.k}$$
$$\ln(2) = k$$
$$A = A_0 e^{kt} = A_0 e^{t\ln(2)}$$
$$A = A_0 (2)^t$$
$$= 3000 \ (2)^2$$
$$= 12000$$

⊗ $\int \sin^2 x \, dx$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$\cos 2\theta = 1 - 2\sin^2\theta$$
$$\cos 2\theta = 2\cos^2\theta - 1$$
$$\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$$
$$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$$

$$= \frac{1}{2}\int 1 - \cos 2\theta \, d\theta$$
$$= \frac{1}{2}\left[\theta - \frac{\sin 2\theta}{2}\right] + C$$
$$= \frac{\theta}{2} - \frac{\sin 2\theta}{4} + C$$
$$= \frac{\theta}{2} - \frac{2\sin\theta\cos\theta}{4} + C$$
$$= \frac{\theta}{2} - \frac{\sin\theta\cos\theta}{2} + C$$

⊛ $\int 2x \tan^{-1} x^2 \, dx$

$= \int \tan^{-1} y \, dy$

$= y \tan^{-1} y - \int \frac{y}{1+y^2} \, dy$

$= y \tan^{-1} y - \frac{1}{2} \int \frac{du}{u}$

$= y \tan^{-1} y - \frac{1}{2} \ln|u| + C$

$= x^2 \tan^{-1} x^2 - \frac{1}{2} \ln|1+x^4| + C$

$y = x^2$

$dy = 2x\,dx$

$u = \tan^{-1} y \qquad dv = dy$

$du = \frac{1}{1+y^2} dy \qquad v = y$

$u = 1+y^2$

$du = 2y\,dy$

⊛ $\int x \ln(2x) \, dx$

$= \frac{x^2}{2} \ln 2x - \int \frac{1}{\cancel{x}} \frac{x^2}{2} \, dx$

$= \frac{x^2}{2} \ln 2x - \frac{1}{2} \int x \, dx$

$= \frac{x^2}{2} \ln 2x - \frac{x^2}{4} + C$

$u = \ln 2x \qquad dv = x\,dx$

$du = \frac{2}{2x} dx$

$du = \frac{1}{x} dx \qquad v = \frac{x^2}{2}$

(*) $\int 6x \ln x^7 \, dx$

$= 42 \int x \ln x \, dx$

(*) $\int \tan^4 8x \, dx$ $\qquad 1 + \tan^2\theta = \sec^2\theta.$

$= \int \tan^2 8x \tan^2 8x \, dx$

$= \int (\sec^2 8x - 1) \tan^2 8x \, dx$

$= \int \sec^2 8x \tan^2 8x \, dx - \int \tan^2 8x \, dx$

$= \int \sec^2 8x \tan^2 8x \, dx - \int \sec^2 8x - 1 \, dx$

$= \int \sec^2 8x \tan^2 8x \, dx - \int \sec^2 8x \, dx + \int dx$

$= \frac{1}{8} \int u^2 du - \frac{\tan 8x}{8} + x$ $\qquad u = \tan 8x$, $du = 8 \sec^2 8x \, dx$

$= \frac{u^3}{24} - \frac{\tan 8x}{8} + x + C$

$= \frac{\tan^3 8x}{24} - \frac{\tan 8x}{8} + x + C$

$\circledast$ $\int \sin^7 2x \cos^4 2x \, dx$

$= \int \sin^6 2x \cos^4 2x \sin 2x \, dx$

$= \int (\sin^2 2x)^3 \cos^4 2x \sin 2x \, dx$

$= \int (1-\cos^2 2x)^3 \cos^4 2x \sin 2x \, dx$

$= -\frac{1}{2}\int (1-u^2)^3 u^4 \, du$

$u = \cos 2x$

$du = -2\sin 2x \, dx$

$= -\frac{1}{2}\int (1-u^6-3u^2+3u^4) u^4 \, du$

$= -\frac{1}{2}\int u^4 - u^{10} - 3u^6 + 3u^8 \, du$

$= -\frac{u^5}{10} + \frac{u^{11}}{22} + \frac{3u^7}{14} - \frac{3u^9}{18} + C$

$= -\frac{\cos^5 2x}{10} + \frac{\cos^{11} 2x}{22} + \frac{3\cos^7 2x}{14} - \frac{\cos^9 2x}{6} + C$

⊛ $\int \cos^3 3x \sin^5 3x \, dx$

$= \int \cos^2 3x \sin^5 3x \cos 3x \, dx$

$= \int (1-\sin^2 3x) \sin^5 3x \cos 3x \, dx$ $\qquad u = \sin 3x$

$du = 3\cos 3x \, dx$

$= \frac{1}{3}\int (1-u^2) u^5 \, du$

$= \frac{1}{3}\int u^5 - u^7 \, du$

$= \frac{u^6}{18} - \frac{u^8}{24} + C = \frac{\sin^6 3x}{18} - \frac{\sin^8 3x}{24} + C$

⊛ $\int_0^{\pi/2} 5\cos^2(10x)\,dx$ $\qquad \cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$

$= \frac{5}{2}\int_0^{\pi/20} (1+\cos(20x))\,dx$

$= \frac{5}{2}\left[x + \frac{\sin 20x}{20}\right]_0^{\pi/20}$

$= \frac{5}{2}\left[\frac{\pi}{20} + \frac{\sin\pi}{20}\right] - 0$

$= \frac{5\pi}{40}$

$$\circledast \quad \int \frac{2}{x^2\sqrt{1+x^2}}\,dx \qquad x = \tan\theta, \quad dx = \sec^2\theta\, d\theta$$

$$= \int \frac{2 \quad \sec^2\theta\, d\theta}{\tan^2\theta\,\sqrt{1+\tan^2\theta}}$$

$$= \int \frac{2\sec^{\cancel{2}}\theta\, d\theta}{\tan^2\theta\,\cancel{\sec\theta}}$$

$$= 2\int \frac{\sec\theta\,\cos^2\theta\, d\theta}{\sin^2\theta} = 2\int \frac{\cos\theta}{\sin^2\theta}\,d\theta$$

$$= 2\int \frac{du}{u^2} \qquad u = \sin\theta, \quad du = \cos\theta\, d\theta$$

$$= -\frac{2}{u} + c = -\frac{2}{\sin\theta} + c = -\frac{2\sqrt{1+x^2}}{x} + c$$

$$\circledast \quad \int \tan^5 x\,\sec^3 x\, dx$$

$$= \int \tan^4 x\,\sec^2 x \cdot \sec x \tan x\, dx$$

$$= \int (\sec^2 x - 1)^2 \sec^2 x \cdot \sec x \tan x\, dx \qquad u = \sec x, \quad du = \sec x \tan x\, dx$$

$$= \int (u^2-1)^2 u^2\, du$$

$$= \int (u^4 + 1 - 2u^2)u^2\, du$$

$$= \int u^6 + u^2 - 2u^4\, du = \frac{u^7}{7} + \frac{u^3}{3} - \frac{2u^5}{5} + c$$

$$= \frac{\sec^7 x}{7} - \frac{2}{5}\sec^5 x + \frac{\sec^3 x}{3} + c$$

(*) $\int \tan^4 \pi x \sec^4 \pi x \, dx.$

$= \int \tan^4 \pi x \cdot \sec^2 \pi x \cdot \sec^2 \pi x \, dx$

$= \int \tan^4 \pi x \, (1 + \tan^2 \pi x) \sec^2 \pi x \, dx$

$= \frac{1}{\pi} \int u^4 (1 + u^2) \, du$

$u = \tan \pi x$

$du = \pi \sec^2 \pi x \, dx$

$= \frac{1}{\pi} \int u^4 + u^6 \, du$

$= \frac{u^5}{5\pi} + \frac{u^7}{7\pi} + C = \frac{\tan^7 \pi x}{7\pi} + \frac{\tan^5 \pi x}{5\pi} + C$

(*) $\int x^3 \cos(8x^2) \, dx$

$y = x^2$

$dy = 2x \, dx$

$= \int x \cdot x^2 \cos(8x^2) \, dx$

$= \frac{1}{2} \int y \cos(8y) \, dy$

$u = y \quad dv = \cos 8y \, dy$

$du = dy \quad v = \frac{\sin 8y}{8}$

$= \frac{1}{2}\left[\frac{y \sin 8y}{8} - \int \frac{\sin 8y}{8} \, dy \right.$

$= \frac{1}{2}\left[\frac{y \sin 8y}{8} + \frac{\cos 8y}{64} \right] + C$

$= \frac{x^2 \sin 8x^2}{16} + \frac{\cos 8x^2}{128} + C$

$$\int \frac{dx}{x^2\sqrt{5-x^2}}$$

$x = \sqrt{5}\sin\theta$

$dx = \sqrt{5}\cos x\, dx$

$$= \int \frac{\sqrt{5}\cos\theta\, d\theta}{5\sin^2\theta\sqrt{5-5\sin^2\theta}}$$

$5 - 5\sin^2\theta$

$= 5(1-\sin^2\theta)$

$= 5\cos^2\theta$

$$= \int \frac{\sqrt{5}\cos\theta\, d\theta}{5\sin^2\theta\sqrt{5\cos^2\theta}}$$

$$= \int \frac{\cancel{\sqrt{5}}\cancel{\cos\theta}\, d\theta}{5\sin^2\theta\,\cancel{\sqrt{5}}\,\cancel{\cos\theta}} = \frac{1}{5}\int \csc^2\theta\, d\theta$$

$$= -\frac{1}{5}\cot\theta + C$$

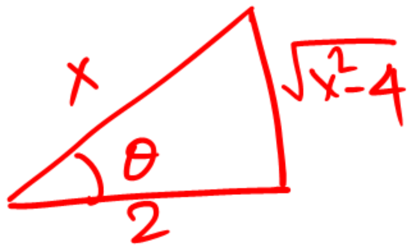
$$= -\frac{1}{5}\frac{\sqrt{5-x^2}}{x} + C$$

$$\int \frac{\sqrt{x^2-4}}{x}\, dx$$

$x = 2\sec\theta$

$dx = 2\sec\theta\tan\theta\, d\theta$

$$= \int \frac{\sqrt{4\sec^2\theta - 4}}{\cancel{2\sec\theta}}\,\cancel{2\sec\theta}\tan\theta\, d\theta$$



$$= \int 2\tan\theta\tan\theta\, d\theta$$

$$= 2\int \tan^2\theta\, d\theta$$

$$= 2\int \sec^2\theta - 1\, d\theta$$

$$= 2\tan\theta - 2\theta + C$$

$x = 2\sec\theta$

$\frac{2}{x} = \cos\theta$

$\theta = \cos^{-1}\frac{2}{x}$

$$= \cancel{2}\frac{\sqrt{x^2-4}}{\cancel{2}} - 2\cos^{-1}\frac{2}{x} + C$$

(*) $$\int \frac{1}{e^{x}\sqrt{9-e^{2x}}}\,dx$$

$$u = e^{x}$$
$$du = e^{x}\,dx$$
$$\frac{du}{e^{x}} = dx$$
$$\frac{du}{u} = dx$$

$$= \int \frac{1}{u\sqrt{9-u^2}}\,\frac{du}{u}$$

$$= \int \frac{du}{u^2\sqrt{9-u^2}}$$

$$u = 3\sin\theta$$
$$du = 3\cos\theta\,d\theta$$

$$= \int \frac{3\cos\theta\,d\theta}{9\sin^2\theta\sqrt{9-9\sin^2\theta}}$$

$$= \int \frac{\cancel{3}\,\cancel{\cos\theta}\,d\theta}{9\sin^2\theta\,\cancel{3}\,\cancel{\cos\theta}} = \frac{1}{9}\int \csc^2\theta\,d\theta$$

$$= -\frac{1}{9}\cot\theta + C$$

$$= -\frac{1}{9}\,\frac{\sqrt{9-u^2}}{u} + C = \frac{-\sqrt{9-e^{2x}}}{9e^{x}} + C$$

$$\int \frac{x \, dx}{\sqrt{x^2 - 2x + 5}}$$

$$= \int \frac{x \, dx}{\sqrt{(x-1)^2 + 4}}$$

$$= \int \frac{(2\tan\theta + 1)\, 2\sec^2\theta \, d\theta}{\sqrt{4\tan^2\theta + 4}}$$

$$= \int \frac{(2\tan\theta + 1)\, \cancel{2}\sec^{\cancel{2}}\theta \, d\theta}{\cancel{2}\sec\cancel{\theta}}$$

$$= 2\int \tan\theta \sec\theta + \int \sec\theta \, d\theta$$

$$= 2\sec\theta + \ln|\sec\theta + \tan\theta| + C$$

$$= 2\frac{\sqrt{x^2-2x+5}}{2} + \ln\left|\frac{\sqrt{x^2-2x+5}}{2} + \frac{x-1}{2}\right| + C$$

$$= \sqrt{x^2-2x+5} + \ln\left|\frac{\sqrt{x^2-2x+5} + x - 1}{2}\right| + C$$

$$x^2 - 2x + 5$$

$$= x^2 - 2\cdot 1 \cdot x + 1 + 4$$

$$= (x-1)^2 + 4$$

$$x - 1 = 2\tan\theta$$

$$dx = 2\sec^2\theta \, d\theta$$

$$x = 2\tan\theta + 1$$