Lab 12/30

(*) $\int 2\tan^5(10x)\,dx$

$= 2\int \tan^4(10x)\tan 10x\,dx$

$= 2\int (\sec^2 10x - 1)^2 \tan 10x\,dx$

$= 2\int (\sec^4 10x + 1 - 2\sec^2 10x)\tan 10x\,dx$

$= 2\int \sec^4 10x \tan x\,dx + 2\int \tan 10x\,dx$

$\qquad - 2\int \sec^2 10x \tan 10x\,dx$

$= 2\int \sec^3 10x \sec 10x \tan x\,dx + 2\int \frac{\sin 10x}{\cos 10x}\,dx$

$\qquad - 2\int \sec 10x \sec 10x \tan 10x\,dx$

$u = \sec 10x \qquad v = \cos 10x$

$du = 10\sec 10x \tan 10x \qquad dv = -10\sin 10x\,dx$

$= \frac{2}{10}\int u^3\,du - \frac{2}{10}\int \frac{1}{v}\,dv - \frac{2}{10}\int u\,du$

$= \frac{2}{40}u^4 - \frac{2}{10}\ln|v| - \frac{1}{5}u^2 + C$

$= \frac{1}{20}\sec^4 10x - \frac{1}{5}\ln|\cos 10x| - \frac{1}{5}\sec^2 10x + C$

$= \frac{1}{20}(1+\tan^2 10x)^2 - \frac{1}{5}\ln\left|\frac{1}{\sec 10x}\right| - \frac{1}{5}(\tan^2 10x + 1) + C$

$= \frac{1}{20}(1+\tan^4 10x + 2\tan^2 10x) - \frac{1}{5}(\ln|1| - \ln|\sec 10x|)$

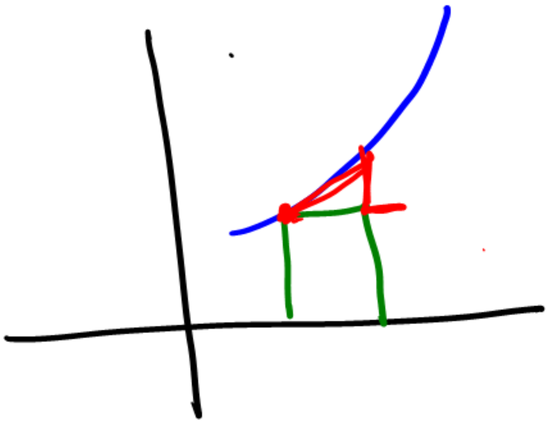
$\qquad - \frac{1}{5}\tan^2 10x - \frac{1}{5} + C$

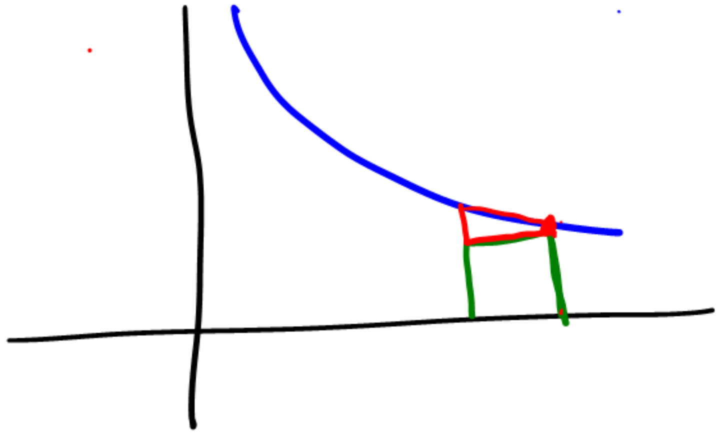
$$= \frac{1}{20} + \frac{1}{20}\tan^4 10x + \frac{2}{20}\tan^2 10x + \frac{1}{5}\ln|\sec 10x|$$
$$- \frac{1}{5}\tan^2 10x - \frac{1}{5} + C$$
$$= \frac{1}{20}\tan^4 10x - \frac{2}{20}\tan^2 10x + \frac{1}{5}\ln|\sec 10x| + C$$
$$= \frac{1}{20}\tan^4 10x - \frac{1}{10}\tan^2 10x + \frac{1}{5}\ln|\sec 10x| + C$$

(*) $2\int \tan^5 10x \, dx$
$$= 2\int \tan^3 10x \,(\tan^2 10x)\, dx$$
$$= 2\int \tan^3 10x\,(\sec^2 10x - 1)\, dx$$
$$= 2\int \tan^3 10x \sec^2 10x\, dx - 2\int \tan^3 10x\, dx$$
$u = \tan 10x$

$du = 10\sec^2 10x\, dx$
$$= \frac{2}{10}\int u^3\, du - 2\int \tan 10x\,(\sec^2 10x - 1)\, dx$$
$$= \frac{1}{5}\frac{u^4}{4}du - 2\int \tan 10x \sec^2 10x\, dx + 2\int \tan 10x\, dx$$
$$= \frac{\tan^4 10x}{20} - \frac{2}{10}\int u\, du + \frac{2}{10}\ln|\sec 10x|$$
$$= \frac{\tan^4 10x}{20} - \frac{1}{10}\tan^2 10x + \frac{1}{5}\ln|\sec 10x| + C$$

(*) $R_n = L_f(P)$





$L_n = L_f(P)$

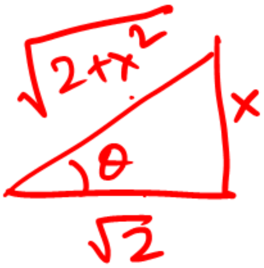
(*) $$\int \frac{3x^2}{\sqrt{2+x^2}}\,dx$$

$x = \sqrt{2}\tan\theta$

$dx = \sqrt{2}\sec^2\theta\, d\theta$

$$= \int \frac{3 \cdot 2\tan^2\theta \ \sqrt{2}\sec^2\theta\, d\theta}{\sqrt{2+2\tan^2\theta}}$$

$$= \int \frac{3\cdot 2\tan^2\theta \cdot \cancel{\sqrt{2}}\sec^{\cancel{2}}\theta\, d\theta}{\cancel{\sqrt{2}}\cancel{\sec}\theta}$$



$$= 6\int \tan^2\theta \sec\theta\, d\theta$$

$$= 6\int (\sec^2\theta - 1)\sec\theta\, d\theta$$

$$= 6\int (\sec^3\theta - \sec\theta)\, d\theta$$

$\sec\theta = \frac{\sqrt{2+x^2}}{\sqrt{2}}$

$\tan\theta = \frac{x}{\sqrt{2}}$

$$= 6\int \sec^3\theta\, d\theta - 6\int \sec\theta\, d\theta$$

$$= 6\left(\frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\sec\theta + \tan\theta|\right) - 6\ln|\sec\theta + \tan\theta| + C$$

$$= 3\sec\theta\tan\theta - 3\ln|\sec\theta + \tan\theta| + C$$

$$= 3\sec\theta\tan\theta - 3\ln|\sec\theta + \tan\theta| + C$$

$$= 3\,\frac{x\sqrt{x^2+2}}{\sqrt{2}\cdot\sqrt{2}} - 3\ln\left|\frac{\sqrt{x^2+2}}{\sqrt{2}} + \frac{x}{\sqrt{2}}\right| + C$$

$$= \frac{3x\sqrt{x^2+2}}{2} - 3\ln\left|\frac{\sqrt{2+x^2}+x}{\sqrt{2}}\right| + C$$

(*) $$\int \frac{2x^2}{\sqrt{64-x^2}}\,dx$$

$x = 8\sin\theta$

$dx = 8\cos\theta\,d\theta$

$$= \int \frac{2\cdot 64\sin^2\theta \cdot 8\cos\theta\,d\theta}{\sqrt{64-64\sin^2\theta}}$$

$$= \int \frac{2\cdot 64\cdot\sin^2\theta\cdot\cancel{8}\,\cancel{\cos}\theta\,d\theta}{\cancel{8}\,\cancel{\cos}\theta}$$

$$\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$$

$$= 128\int \sin^2\theta\,d\theta$$

$$= 64\int 1-\cos 2\theta\,d\theta = 64\left[\theta - \frac{\sin 2\theta}{2}\right] + C$$

$$= 64\theta - 64\cdot\frac{\cancel{2}\cdot\sin\theta\cos\theta}{\cancel{2}} + C$$

$$= 64\sin^{-1}\frac{x}{8} - \cancel{64}\,\frac{x}{\cancel{8}}\,\frac{\sqrt{64-x^2}}{\cancel{8}} + C$$

$$= 64\sin^{-1}\frac{x}{8} - x\sqrt{64-x^2} + C$$

(*) $\displaystyle\int \frac{2x^2}{(x^2+3)^{3/2}}\,dx$

$x = \sqrt{3}\tan\theta$

$dx = \sqrt{3}\sec^2\theta\, d\theta$

$$= \int \frac{2\cdot 3\cdot\tan^2\theta\cdot\sqrt{3}\sec^2\theta\, d\theta}{(3\tan^2\theta+3)^{3/2}}$$

$$(3\tan^2\theta+3)^{3/2}$$
$$= (3(\tan^2\theta+1))^{3/2}$$
$$= 3^{3/2}(\sec^2\theta)^{3/2}$$
$$= 3\sqrt{3}\sec^3\theta$$

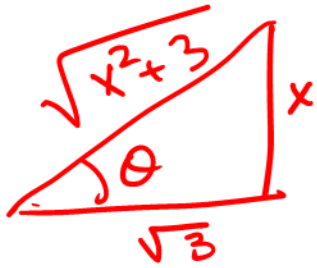
$$= \int \frac{2\cdot\cancel{3}\tan^2\theta\,\cancel{\sqrt{3}}\,\cancel{\sec^2}\theta\, d\theta}{\cancel{3}\cancel{\sqrt{3}}\sec^{\cancel{3}}\theta}$$

$$= 2\int \frac{\tan^2\theta}{\sec\theta}\,d\theta$$

$$= 2\int \frac{\sec^2\theta - 1}{\sec\theta}\,d\theta$$

$$= 2\int(\sec\theta - \cos\theta)\,d\theta$$

$$= 2\ln|\sec\theta+\tan\theta| - 2\sin\theta + C$$



$$= 2\ln\left|\frac{\sqrt{x^2+3}}{\sqrt{3}} + \frac{x}{\sqrt{3}}\right| - 2\frac{x}{\sqrt{x^2+3}} + C$$

(*) $\int \sec^3 x\,dx = \int \sec x \sec^2 x\,dx$

$u = \sec x \qquad dv = \sec^2 x\,dx$

$du = \sec x \tan x \qquad v = \tan x$

$\int \sec^3 x\,dx = \sec x \tan x - \int \tan^2 x \sec x\,dx$

$= \sec x \tan x - \int (\sec^2 x - 1)\sec x\,dx$

$\int \sec^3 x\,dx = \sec x \tan x - \int \sec^3 x\,dx + \int \sec x\,dx$

$2\int \sec^3 x\,dx = \sec x \tan x + \ln|\sec x + \tan x| + C$

$\int \sec^3 x\,dx = \frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x| + C$

(*) $\int_4^5 \sin(4x)$ $\varepsilon = 0.01$

smalles n?

1
2
3
4
5

$$|E_n^T| \le \frac{(b-a)^3}{12n^2} \max|f''(x)|$$

$a=4$ $b=5$

$f(x) = \sin 4x$

$f'(x) = 4\cos 4x$

$f''(x) = -16\sin 4x$

$|f''(x)| = |-16\sin 4x|$

$= 16|\sin 4x| \le 16 \cdot 1$

$= 16$

$$\frac{(b-a)^3}{12n^2} \max|f''(x)| \le 0.01$$

$$\frac{(5-4)^3}{3\,\cancel{12}\,n^2} \cdot \cancel{16}^{\,4} \le \frac{1}{100}$$

$$n^2 \geq \frac{4 \cdot 100}{3}$$

$$n^2 \geq 133.33$$

$$n = 12$$

$n=10$ $n^2=100$

$n=11$ $n^2=121$

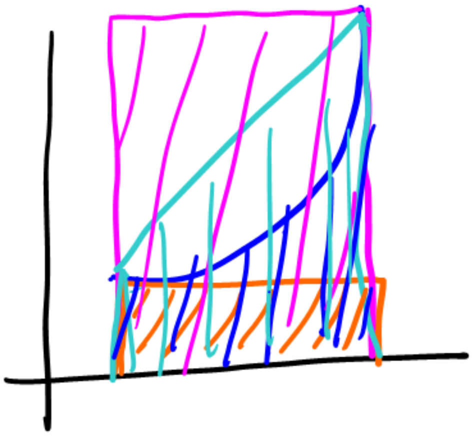
$n=12$ $n^2=144$

$n=13$ $n^2=169$

(*) L, R, S, T

- f(x) concave up increasin

$$L < \int f(x)dx < T < R$$



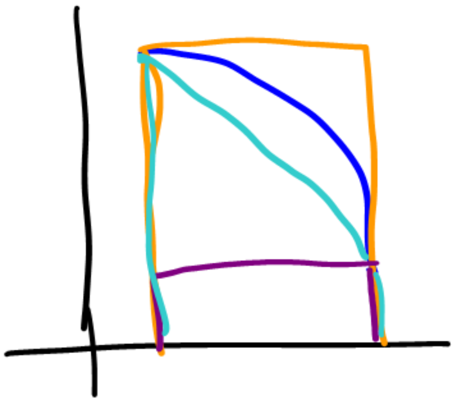
$$L < \int f(x)dx < S < R$$

- f(x) concav down decreasing

$$L > \int f(x)dx > T > R$$

overestimets

under estimets



Any case Simpson will give best approx. and least error.

(*) $$\frac{6x^3+3x+3}{x^6-21x^4-100x^2} = \frac{6x^3+3x+3}{x^2(x^4-21x^2-100)}$$

$u = x^2$

$u^2 = x^4$

$x^4 - 21x^2 - 100$

$= u^2 - 21u - 100$

$= u^2 - 25u + 4u - 100$

$= u(u-25) + 4(u-25)$

$= (u-25)(u+4)$

$= (x^2-25)(x^2+4) = (x-5)(x+5)(x^2+4)$

$$\frac{6x^2+3x-3}{x^2(x^4-21x-100)}$$

$$= \frac{6x^2+3x-3}{x^2(x-5)(x+5)(x^2+4)}$$

$$\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-5} + \frac{D}{x+5} + \frac{Cx+D}{x^2+4}$$

(*) $\displaystyle\int \frac{4x^2-7x+9}{(x^2+1)(x-6)}\,dx$

$$\frac{4x^2-7x+9}{(x^2+1)(x-6)} = \frac{Ax+B}{x^2+1} + \frac{C}{x-6}$$

$$4x^2-7x+9 = (Ax+B)(x-6) + C(x^2+1)$$

$x=6$ $\quad 4\cdot 36-42+9 = (A\cdot 6+B)(6-6) + C(36+1)$

$$111 = 37C \Rightarrow C = \frac{111}{37} = 3$$

$x=0$ $\quad 9 = (A\cdot 0+B)(0-6) + 3(0+1)$

$$9 = -6B+3 \Rightarrow B = -1$$

$x=1$ $\quad 4-7+9 = (A-1)(-5) + 3(2)$

$$6 = -5A+5+6$$

$$6 = -5A+11 \Rightarrow -5 = -5A$$

$$A = 1$$

$$\int \frac{4x^2-7x+9}{(x^2+1)(x-6)}\,dx = \int \frac{x-1}{x^2+1}\,dx + \int \frac{3}{x-6}\,dx$$

$$= \int \frac{x}{x^2+1}\,dx - \int \frac{1}{x^2+1}\,dx + \int \frac{3}{x-6}\,dx$$

$$= \frac{1}{2}\ln|x^2+1| - \tan^{-1}x + 3\ln|x-6| + C$$

(*) $$\int \frac{x^3}{1+x^2}\,dx$$

$$= \int \frac{x^3+x-x}{1+x^2}\,dx$$

$$= \int \frac{x^3+x}{1+x^2}\,dx - \int \frac{x}{1+x^2}\,dx$$

$$= \int \frac{x\cancel{(x^2+1)}}{\cancel{(1+x^2)}}\,dx - \int \frac{x}{1+x^2}\,dx$$

$$= \frac{x^2}{2} - \frac{1}{2}\ln|1+x^2| + C$$