Lab 12/31

⊛ $\int_0^3 \sin(2x)\,dx$ $\varepsilon = 0.003$

$|E_n^s| \le \frac{(b-a)^5}{180n^4} M$ $M = \max|f^4(x)|$

$a=0\ b=3$

$f(x) = \sin 2x$

$f'(x) = 2\cos 2x$

$f''(x) = -2^2 \sin 2x$

$f'''(x) = -2^3 \cos 2x$

$f^4(x) = 2^4 \sin 2x$

$\therefore |f^4(x)| = |2^4 \sin 2x| \le 16 = M$

$\frac{3^5}{180n^4} 16 \le 0.003$

$\frac{\overset{9}{\cancel{81}} \cdot \cancel{3}\ \overset{8}{\cancel{16}}}{\cancel{9}\ \cancel{180}\ n^4} \le \frac{\cancel{3}}{10\cancel{0}}$

$n=5$ $n^4 = 625$

$n=6$ $n^4 = 1296$

$n^4 \geq 9.8$

$n^4 \geq 720$

$n=6$

Chap. 9

$\{-2, 0, 1, 2, 3\}$

Upper bound = $3, 4, 5, 100, 10^5$

Lower bound = $-2, -3, -50, -10^6$

LUB = 3 GLB = -2

UB & LB ⇒ Bounded

Not bounded ⇒ Not convgt

But Not convgt $\not\Rightarrow$ Not bounded

There can exist bounded sequences which will not be convg.

	GLB	LUB
$\{x : x^2 < 9\}$ $x^2 < 9 \Rightarrow -3 \leq x \leq 3$	-3	3
$\{x : x^3 \leq 1\}$	DNE	1
$\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots\}$	0	1
$\{-1, -\frac{1}{4}, -\frac{1}{9}, -\frac{1}{16}, \dots\}$	-1	0

(*) $\left\{ \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \dots \right\}$

1 2 3 4 - .

$$a_n = \frac{n}{n+1}$$

$\left\{ 2, -\frac{5}{2}, \frac{10}{3}, -\frac{17}{4}, \frac{26}{5}, \dots \right\}$

1 2 3 4 5 - .

$$a_n = \frac{(-1)^{n+1}\, n^2+1}{n}$$

$(-1)^2 \; (-1)^4 \; (-1)^6 = 1$

$(-1)^3 \; (-1)^5 \; (-1)^7 = -1$

(*) 1, 2, 3, 4, 5, - - . mono inc.

$-1, -\frac{1}{2}, -\frac{1}{3}, -\frac{1}{4}, \dots$ mono dec.

1, 0, 2, 3, 4, 5 - - . NOT Mono.

(*) $a_n = \frac{3n+2}{4n}$

GLB = 3/4, LUB = 5/4 } Bdd.

$\frac{5}{4}, \frac{8}{8}, \frac{11}{12}, \frac{14}{16}, \frac{17}{20}$

$$a_n = \frac{3\cancel{n}}{4\cancel{n}} + \frac{2}{\cancel{4}n}$$

$$= \frac{3}{4} + \boxed{\frac{1}{2n}} \to 0$$

$n \to \infty$

$$a_n = \frac{3n+2}{4n} = \frac{3}{4} + \underline{\underline{\frac{1}{2n}}}$$ Mono dec

(*) $$a_n = \frac{3^n}{2^n+90}$$

GLB = $\frac{3}{92}$

LUB = DNE

$$\frac{3}{92}, \frac{9}{94}, \frac{27}{98}, \frac{81}{106}, \frac{405}{122}, \ldots$$

$$\frac{a_{n+1}}{a_n} = \frac{3^{n+1}}{2^{n+1}+90} \cdot \frac{2^n+90}{3^n}$$

$$= \frac{\cancel{3^n}\cdot 3}{\cancel{3^n}} \cdot \frac{2^n+90}{2(2^n+45)}$$

$$= \frac{3}{2}\left(\frac{2^n+90}{2^n+45}\right) > 1$$

>1

$$\left(\frac{2^n+45+45}{2^n+45}\right)$$

$$\left(\frac{2^n+45}{2^n+45} + \frac{45}{2^n+45}\right)$$

$$\left(1+\frac{45}{2^n+45}\right)$$

$$\frac{a_{n+1}}{a_n} > 1$$

$a_{n+1} > a_n$

Mono Inc

⊛ $$\lim_{n\to\infty} \frac{n+(-1)^n}{2n} = \lim_{n\to\infty} \frac{1}{2} + \frac{(-1)^n}{2n}$$

$$= \frac{1}{2}$$

⊛ $$\lim_{n\to\infty} \frac{2n+3}{5n+2} = \frac{2}{5}$$

⊛ $$\lim_{n\to\infty} \frac{2n^2+3}{5n+2} = \text{DNE}$$

⊛ $$\lim_{n\to\infty} \frac{2n+3}{5n^2+2} = 0$$

⊛ $$\lim_{n\to\infty} \frac{2^n}{3^n+100} = \lim_{n\to\infty} \frac{\left(2/3\right)^n \to 0}{\underline{1} + \boxed{\frac{100}{3^n}} \to 0} = 0 \qquad \frac{2}{3} < 1$$

⊛ $$\lim_{n\to\infty} \tan \frac{n\pi}{4n+2}$$

$$= \tan\left(\lim_{n\to\infty} \frac{n\pi}{4n+n}\right)$$

$$= \tan\left(\frac{\pi}{4}\right)$$

$$= 1$$

⊛ $\lim\limits_{n\to\infty} \frac{5^{n-2}}{7^{n+1}} = \lim\limits_{n\to\infty} \frac{5^{n}.5^{-2}}{7^{n}.7}$

$= \lim\limits_{n\to\infty} \left(\frac{5}{7}\right)^{n} \frac{1}{25.7}$

$= 0$

⊛ $\lim\limits_{n\to\infty} \frac{5^{2n-2}}{7^{n+1}} = \lim\limits_{n\to\infty} \left(\frac{5^{2}}{7}\right)^{n} \frac{1}{25.7}$

$= \lim\limits_{n\to\infty} \left(\frac{25}{7}\right)^{n} \frac{1}{25.7}$

$= \text{DNE}$

⊛ $\lim\limits_{n\to\infty} \left(1+\frac{1}{n}\right)^{2n} = e^{2}$ $\qquad \lim\limits_{n\to\infty} \left(1+\frac{a}{n}\right)^{bn} = e^{ab}$

⊛ $\lim\limits_{n\to\infty} \left(\frac{n}{n+1}\right)^{2n}$

$= \lim\limits_{n\to\infty} \left(\frac{n+1}{n}\right)^{-2n}$

$= \lim\limits_{n\to\infty} \left(1+\frac{1}{n}\right)^{-2n} = e^{-2}$

⊛ $\sum \left(\frac{k}{k+1}\right)^{k}$ BDT $a_k \not\to 0$ $\sum a_k$ div

$a_k = \left(\frac{k}{k+1}\right)^{k} \to e^{-1} \neq 0$

⊛ $\lim_{n\to\infty} n \sin \frac{2}{n}$

$= \lim_{n\to\infty} \frac{\sin \frac{2}{n}}{\frac{1}{n}}$

$h = \frac{1}{n}$

as $n \to \infty$, $h \to 0$

$= \lim_{h\to 0} \frac{\sin 2h}{h}$

$\lim_{x\to 0} \frac{\sin ax}{bx} = \frac{a}{b}$

$= 2$

⊛ $\lim_{n\to\infty} \sqrt{n^2+2n} - n$

$= \lim_{n\to\infty} \frac{(\sqrt{n^2+2n} - n)(\sqrt{n^2+2n} + n)}{\sqrt{n^2+2n} + n}$

$= \lim_{n\to\infty} \frac{(n^2+2n) - (n^2)}{\sqrt{n^2+2n} + n} = \lim_{n\to\infty} \frac{2n}{\sqrt{n^2+2n} + n}$

$= \lim_{n\to\infty} \frac{\frac{2n}{n}}{\frac{\sqrt{n^2+2n}}{n} + \frac{n}{n}}$

$= \lim_{n\to\infty} \frac{2}{\sqrt{\frac{n^2}{n^2} + \frac{2n}{\cancel{n}^2}} + 1}$

$= \lim_{n\to\infty} \frac{2}{\sqrt{1 + \frac{2}{n}} + 1}$

$= 1$

$$\sum_{n=1}^{\infty} a_n.$$

$$\sum_{n=1}^{10} a_n$$

$S_1 = a_1$

$S_2 = a_1 + a_2$

$S_3 = a_1 + a_2 + a_3$

$\vdots$

$S_n = a_1 + a_2 + \dots + a_n$

$$\begin{array}{r} 5.5 \\ 0.75 \\ \hline 6.25 \end{array}$$

$$\sum_{1}^{\infty} \frac{3}{n}$$

$S_1 = 3 = 3$

$S_2 = 3 + \frac{3}{2} = 3 + 1.5 = 4.5$

$S_3 = 3 + \frac{3}{2} + \frac{3}{3} = 4.5 + 1 = 5.5$

$S_4 = 3 + \frac{3}{2} + \frac{3}{3} + \frac{3}{4} = 6.25$

$S_5 = 3 + \frac{3}{2} + \frac{3}{3} + \frac{3}{4} + \frac{3}{5}$

$= 6.25 + 0.6$

$= 6.85$

(*) $$\lim_{n \to \infty} \frac{n^2}{\sqrt{11n^4 + 1}}$$

$$= \lim_{n \to \infty} \frac{\frac{n^2}{n^2}}{\frac{\sqrt{11n^4+1}}{n^2}} = \lim_{n \to \infty} \frac{1}{\sqrt{\frac{11\cancel{n^4}}{\cancel{n^4}} + \underbrace{\frac{1}{n^4}}_{\to 0}}}$$

$$= \frac{1}{\sqrt{11}}$$