Lab 01/07

(*) $\frac{1}{18} - \frac{1}{27} + \cdots + \frac{(-1)^k}{9k} + \cdots$

$$\sum_{k=1}^{\infty} \left| \frac{(-1)^k}{9k} \right| = \sum \frac{1}{9k}$$

conv g

Condition conv g $\Leftarrow$ Abs conv g

Condition conv g $\not\Rightarrow$ Abs conv g

$\sum \frac{1}{9k}$ is div by p-series test as $p=1$

so the series is not abs conv g

$\sum \frac{(-1)^k}{9k}$

(i) mono dec

(ii) $\lim_{k\to\infty} \frac{1}{9k} = 0$

so by Alt Series test the series is conv g

The series is cond conv g

(*) $\sum_{k=1}^{\infty} \left| \frac{(-1)^k \ln k}{k+5} \right| = \sum \frac{\ln k}{k+5}$

$k+5 < k+k$

$\frac{1}{k+5} > \frac{1}{2k}$

$x > \ln x > 1$

$$\sum \frac{\ln k}{k+5} \geq \sum \frac{\ln k}{2k} \geq \sum \frac{1}{2k}$$

$\sum \frac{1}{2k}$ is div by p series $p=1$

$\therefore \sum \frac{\ln k}{k+5}$ is div by DCT.

Hence $\sum \frac{(-1)^k \ln k}{k+5}$ is not abs conv g

$$\sum \frac{(-1)^k \ln k}{k+5}$$

(i) $\frac{\ln k}{k+5}$ is mono dec

$x > \ln x$

$1 > \frac{\ln x}{x}$

(ii) $\lim_{k\to\infty} \frac{\ln k}{k+5} = \lim_{k\to\infty} \frac{1}{k} = 0$

$\therefore$ The series is conv by Alt series test

$\therefore$ It is Cond conv

(*) $\sum \left|\frac{(-1)^k k^3}{4^k}\right| = \sum \frac{k^3}{4^k}$ $a_k = \frac{k^3}{4^k}$

$$\frac{a_{k+1}}{a_k} = \frac{(k+1)^3}{4^{k+1}} \cdot \frac{4^k}{k^3} = \frac{\cancel{4^k}}{\cancel{4^k}\cdot 4}\left(\frac{k+1}{k}\right)^3 \to \frac{1}{4} \text{ as } k\to\infty$$

$\therefore$ The series conv by Ratio test

Hence the series is abs conv

(*) $\sum \left|\frac{(-1)^k k e^{-2k}}{3}\right| = \sum \frac{k e^{-2k}}{3}$

$a_k = \frac{k}{e^{2k}}$

$(a_k)^{1/k} = \frac{k^{1/k}}{e^2} \to \frac{1}{e^2} < 1$ as $k \to \infty$

$2 < e < 3$

The series is conv by root test

Hence the series is abs conv.

$$\frac{a_{k+1}}{a_k} = \frac{k+1}{e^{2k+2}} \cdot \frac{e^{2k}}{k} = \frac{\cancel{e^{2k}}}{\cancel{e^{2k}}\cdot e^2}\left(\frac{k+1}{k}\right) \to \frac{1}{e^2} \text{ as } k\to\infty$$

(*) $\sum \sin\left(\frac{k\pi}{3}\right) = \begin{cases} \frac{\sqrt{3}}{2} \\ 0 \\ -\frac{\sqrt{3}}{2} \end{cases}$ $\quad \cos(\pi k) = \begin{cases} 1 \\ -1 \end{cases}$

(*) $\sum \frac{(-1)^k \cos(\pi k)}{6k+5} = \sum \frac{(-1)^k(-1)^k}{6k+5}$

div by DCT

$= \sum \frac{(-1)^{2k}}{6k+5}$

$= \sum \frac{1}{6k+5} \geq \sum \frac{1}{7k}$ div p-series

(*) $\sum \frac{\cos(\pi k)}{6k+5} = \sum \left|\frac{(-1)^k}{6k+5}\right| = \sum \frac{1}{6k+5}$

$\sum \frac{1}{6k+5} \geq \sum \frac{1}{7k} \to$ div by p-series test

So the series is not abs conv

$\sum \frac{(-1)^k}{6k+5}$ (i) is mono de

(ii) $\lim_{k\to\infty} \frac{1}{6k+5} = 0$

conv by AST. Hence Cond Conv

(*) $\sum \left| \frac{(-1)^n 6n^{5/2} + 3n - 1}{7n^3 + 4n^2 + n} \right|$ $\quad \frac{5}{2} - 3 = -\frac{1}{2}$

$= \sum \frac{6n^{5/2} + 3n - 1}{7n^3 + 4n^2 + n} = \sum a_n \quad \sum b_n = \sum \frac{1}{\sqrt{n}}$

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{6n^3 + 3n^{3/2} - n^{1/2}}{7n^3 + 4n^2 + n}$

$= \frac{6}{7}$

$\sum b_n$ is div by p-series test $p = \frac{1}{2} < 1$

$\sum a_n$ is div by LCT

Hence the series is not abs conv

$\sum \frac{(-1)^n 6n^{5/2} + 3n - 1}{7n^3 + 4n^2 + n}$ (i) mono dec

(ii) $\lim \left(\frac{6n^{5/2} + 3n - 1}{7n^3 + 4n^2 + n} \right) = 0$

So the series conv by AST

Hence it is cond conv

(*) $\sum \frac{(-1)^n 3^{4n}}{4^{3n+1}} = \sum \frac{(-1)^n \cdot 81^n}{64^n \cdot 4}$

$= \frac{1}{4} \sum \left(\frac{-81}{64}\right)^n$

$r = -\frac{81}{64}$ $|r| = \frac{81}{64} > 1 \Rightarrow$ Div by G.S test

(*) $\frac{9x^2}{1+x}$

$\frac{1}{1-x} = \sum x^n$

$= \frac{9x^2}{1-(-x)}$

$= 9x^2 \sum (-x)^n$

$= 9x^2 \sum (-1)^n x^n$

$= 9 \sum (-1)^n x^{n+2}$

(*) $\frac{d}{dx} \sum \frac{(-1)^n (x-2)^{n+2}}{(n+2)!}$

$= \sum \frac{(-1)^n}{(n+2)!} (n+2)(x-2)^{n+1}$

$= \sum \frac{(-1)^n (x-2)^{n+1}}{(n+1)!}$

$\frac{n}{n!}$

$= \frac{\cancel{n}}{1 \cdot 2 \cdots (n-1) \cdot \cancel{n}}$

$= \frac{1}{(n-1)!}$

$\circledast$ $F(x) \int \sum \frac{(-1)^n (x-8)^n}{(n+1)} dx$

$$= \sum \frac{(-1)^n}{(n+1)} \int (x-8)^n dx$$

$$= \sum \frac{(-1)^n}{(n+1)} \frac{(x-8)^{n+1}}{n+1} + C$$

$$= \sum \frac{(-1)^n (x-8)^{n+1}}{(n+1)^2} + C$$

$F(8) = 0 \implies C = 0$

$\circledast$ $\sum_0^\infty \frac{5}{9^k} = 5 \sum_0^\infty \left(\frac{1}{9}\right)^k$ $r = \frac{1}{9}$ $|r| = \frac{1}{9} < 1$

$$= 5 \frac{1}{1-\frac{1}{9}}$$ $\frac{a}{1-r}$

$$= \frac{5}{8/9} = \frac{45}{8}$$

$\circledast$ $\sum_2^\infty \frac{3}{k^2+k} = 3 \sum_2^\infty \frac{1}{k(k+1)} = 3\left[\sum_\infty \frac{A}{k} + \sum \frac{B}{k+1}\right]$

$$= 3 \sum_2^\infty \frac{k+1-k}{k(k+1)} = 3 \sum_2 \left(\frac{1}{k} - \frac{1}{k+1}\right)$$

$$= 3\left[\left(\frac{1}{2} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{4}}\right) + \left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{5}}\right) + \cdots\right]$$

$$= 3/2$$

* $\sum \frac{(-1)^n (x-8)^n}{n^n}$ $\qquad a_n = \frac{(-1)^n (x-8)^n}{n^n}$

$$|(a_n)^{1/n}| = \left| \left(\frac{(-1)^n (x-8)^n}{n^n} \right)^{1/n} \right|$$

$$= \frac{|x-8|}{n} \to 0 \quad \text{as } n \to \infty$$

$R = \infty$ $\quad$ Intv $(-\infty, \infty)$

* $\sum \frac{k^3 (x-7)^k}{e^k}$ $\qquad a_k = \frac{k^3 (x-7)^k}{e^k}$

$$\left|\frac{a_{k+1}}{a_k}\right| = \left| \frac{(k+1)^3 (x-7)^{k+1}}{e^{k+1}} \cdot \frac{e^k}{k^3 (x-7)^k} \right|$$

$$= \left| \left(\frac{k+1}{k}\right)^3 \frac{\cancel{e^k}}{\cancel{e^k} \cdot e} \frac{\cancel{(x-7)^k} \cdot (x-7)}{\cancel{(x-7)^k}} \right|$$

$$\xrightarrow{k \to \infty} \frac{|x-7|}{e} \qquad R = e$$

$$\frac{|x-7|}{e} < 1$$

$$|x-7| < e$$

$$-e < x-7 < e$$

$$7-e < x < 7+e$$

$$\sum \frac{k^3 (x-7)^k}{e^k}$$

$x = 7-e$ $\quad \sum \frac{k^3 (\cancel{7}-e-\cancel{7})^k}{e^k} = \sum \frac{k^3 (-e)^k}{e^k}$

$$= \sum \frac{k^3 (-1)^k \cancel{e^k}}{\cancel{e^k}}$$

$$= \sum (-1)^k k^3$$

Divergent.

$x = 7+e$ $\quad \sum \frac{k^3 (7+e-7)^k}{e^k} = \sum \frac{k^3 e^k}{e^k}$

$$= \sum k^3 \quad \text{Div}$$

Intv $(7-e, 7+e)$