Lab 01/08

$$\frac{6x}{1-x^2} \qquad \frac{1}{1-x} = \sum x^n$$

$$= 6x \sum (x^2)^n$$

$$= 6x \sum x^{2n} = 6\sum x^{2n+1}$$

$$\sum_0^\infty \frac{f^n(a)}{n!}(x-a)^n$$

$$f(a) + \frac{f'(a)}{1!}(x-a)^1 + \frac{f^2(a)}{2!}(x-a)^2 + \cdots$$

$a=0$ $\quad f(0) + \frac{f'(0)}{1!}x + \frac{f^2(0)}{2}x^2 + \cdots$

$$f(4) + \frac{f'(4)}{1!}(x-4) + \frac{f^2(4)}{2!}(x-4)^2 + \frac{f^3(4)}{3!}(x-4)^3 + \frac{f^4(4)}{4!}(x-4)^4$$

$$-1 + \frac{4}{1}(x-4) + \frac{9}{2!}(x-4)^2 + \frac{-5}{3!}(x-4)^3 + \frac{-2}{4!}(x-4)^4$$

$$-1 + 4(x-4) + \frac{9}{2}(x-4)^2 - \frac{5}{6}(x-4)^3 - \frac{1}{12}(x-4)^4$$

⊛ $\sum \frac{(-1)^k k^2 (x+1)^k}{(k+2)!}$

$$\left| \frac{(-1)^{k+1}(k+1)^2(x+1)^{k+1}}{(k+3)!} \frac{(k+2)!}{(-1)^k k^2 (x+1)^k} \right|$$

$$= \left(\frac{k+1}{k}\right)^2 \frac{(k+2)!}{(k+3)!} |x+1|$$

$$= \left(\frac{k+1}{k}\right)^2 \frac{|x+1|}{(k+3)} \xrightarrow{k\to\infty} 0$$

$R = \infty$ Intv $(-\infty, \infty)$

⊛ $\sum \frac{x^k}{(k+3)2^k}$

$$\left| \left(\frac{x^k}{(k+3)2^k}\right)^{1/k} \right| = \frac{|x|}{(k+3)^{1/k} 2} \xrightarrow{k\to\infty} \frac{|x|}{2}$$

$R = 2$ $\frac{|x|}{2} < 1$

$\Rightarrow |x| < 2 \Rightarrow -2 < x < 2$

$x = 2$ $\sum \frac{2^k}{(k+3)2^k} = \sum \frac{1}{k+3} \geq \sum \frac{1}{2k}$ Div

$x = -2$ $\sum \frac{(-2)^k}{(k+3)2^k} = \sum \frac{(-1)^k \cancel{2^k}}{(k+3)\cancel{2^k}} = \sum \frac{(-1)^k}{(k+3)}$

Cong by AST

Intv $[-2, 2)$

$\circledast$ $\sum \frac{(k+2)\, x^{k+4}}{(k+3)}$

$$\left| \frac{(k+3)}{(k+4)} x^{k+5} \cdot \frac{(k+3)}{(k+2)\, x^{k+4}} \right|$$

$$= \frac{(k+3)^2}{(k+4)(k+2)} |x| \xrightarrow{k\to\infty} |x| \qquad R = 1$$

$$|x| < 1 \Rightarrow -1 < x < 1$$

$x = 1$ $\qquad \sum \frac{k+2}{k+3}$ Div

$x = -1$ $\qquad \sum \frac{(k+2)}{(k+3)} (-1)^{k+4} = \sum \frac{k+2}{k+3} (-1)^k (-1)^4$

$$= \sum \frac{(-1)^k\, k+2}{k+3} \quad \text{Div}$$

$(-1, 1)$

$\circledast$ $\sum \frac{(-1)^k (x-3)^k}{k^k}$

$$\left| \left(\frac{(-1)^k (x-3)^k}{k^k} \right)^{1/k} \right| = \frac{|x-3|}{k} \xrightarrow{k\to\infty} 0$$

$R = \infty \qquad (-\infty, \infty)$

(x) $\sum \frac{6^k x^{2k}}{(k+5)^2}$

$$\left|\left(\frac{6^k \cancel{x}^{2k}}{(k+5)^2}\right)^{1/k}\right| = \frac{6 \ x^2}{(k+5)^{2/k}} \xrightarrow{k\to\infty} 6x^2$$

$$6x^2 < 1 \Rightarrow x^2 < \frac{1}{6} \Rightarrow -\frac{1}{\sqrt{6}} < x < \frac{1}{\sqrt{6}}$$

$x = \frac{1}{\sqrt{6}}$ $\sum \frac{6^k \left(\frac{1}{\sqrt{6}}\right)^{2k}}{(k+5)^2} = \sum \frac{\cancel{6^k} \frac{1}{\cancel{6^k}}}{(k+5)^2}$

$= \sum \frac{1}{(k+5)^2}$ conv g

$x = -\frac{1}{\sqrt{6}}$ $\sum \frac{6^k \left(-\frac{1}{\sqrt{6}}\right)^{2k}}{(k+5)^2} = \sum \frac{1}{(k+5)^2} \leq \sum \frac{1}{4k^2}$

conv g by p-series

Intv $\left[-\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right]$

(*) $f(x) = x^3 \cos\left(\frac{x}{2}\right)$ $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$

$$f(x) = x^3 \left[1 - \frac{\left(\frac{x}{2}\right)^2}{2!} + \frac{\left(\frac{x}{2}\right)^4}{4!} - \frac{\left(\frac{x}{2}\right)^6}{6!} + \cdots\right]$$

$$= x^3 - \frac{x^5}{2^2 2!} + \frac{x^7}{2^4 2!} - \frac{x^9}{2^6 6!}$$

$x^3 \to 3x^2 \to 6x \to 6 \to 0 \to 0$
$\quad 1 \quad 2 \quad 3 \quad 4 \quad 5$

$x^5 \to 5x^4 \to 5.4x^3 \to 5.4.3x^2 \to 5.4.3.2x \to \frac{5.4.3.2.1}{5}$ $5!$
$\quad 1 \quad 2 \quad 3 \quad 4 \quad 5$

$x^7 \to 7x^6 \to 7.6x^5 \to 7.6.5x^4 \to 7.6.5.4x^3 \to 7.6.5.4.3x^2$
$\quad 1 \quad 2 \quad 3 \quad 4 \quad 5$

$$f^5(x) = -\frac{5!}{2^2 2!} + \frac{\bigcirc x^2}{2^4 2!} - \frac{\bigcirc x^4}{2^6 6!} + \cdots$$

$$f^5(0) = -\frac{5!}{2^2 2!}$$

$f^4(0) = 0$

$f^6(0) =$

(*) $f(x) = \sin(10x)$ $\sin x = \sum \frac{(-1)^n x^{2n+1}}{(2n+1)!}$

$$= 10x - \frac{10^3 x^3}{3!} + \frac{10^5 x^5}{5!} + \cdots$$

$= x - \frac{x^3}{3!} + \frac{x^5}{5!} -$

$$= \sum \frac{(-1)^n (10)^{2n+1} x^{2n+1}}{(2n+1)!}$$

(*) $\sum (k+4)!\,(x-2)^k$

$$\left|\frac{(k+5)!\,(x-2)^{k+1}}{(k+4)!\,(x-2)^k}\right| = (k+5)|x-2| \xrightarrow{k\to\infty} \infty$$

$R = 0$

only convrg when $x-2=0 \Rightarrow x=2$ $\{2\}$

(*) $f(x) = 3\sqrt{1+x}$ $\qquad f(0) = 3$

$f'(x) = \frac{3}{2}(1+x)^{-1/2}$ $\qquad f'(0) = \frac{3}{2}$

$f''(x) = -\frac{3}{4}(1+x)^{-3/2}$ $\qquad f''(0) = -\frac{3}{4}$

$f'''(x) = \frac{9}{8}(1+x)^{-5/2}$ $\qquad f^3(0) = \frac{9}{8}$

$f^4(x) = -\frac{45}{16}(1+x)^{-7/2}$ $\qquad f^4(0) = -\frac{45}{16}$

$$P_4(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \frac{f^4(0)}{4!}x^4$$

$$= 3 + \frac{3}{2}x - \frac{3}{8}x^2 + \frac{9}{48}x^3 - \frac{45}{384}x^4$$

(*) $\sum \sin\left(\frac{k\pi}{3}\right) = \begin{cases} \pi/3 \\ 0 \\ -\pi/3 \end{cases}$

$\sin\frac{\pi}{3} = \sin\frac{7\pi}{3} = \sin\frac{13\pi}{3} - \cdots$

$\sin\frac{2\pi}{3} = \sin\frac{8\pi}{3} = - - \cdot$

$\sum \sin\left(\frac{k\pi}{11}\right)$ $\quad \sin\frac{\pi}{11} = \sin\frac{23\pi}{11} = \sin\frac{45\pi}{11} = \cdots$

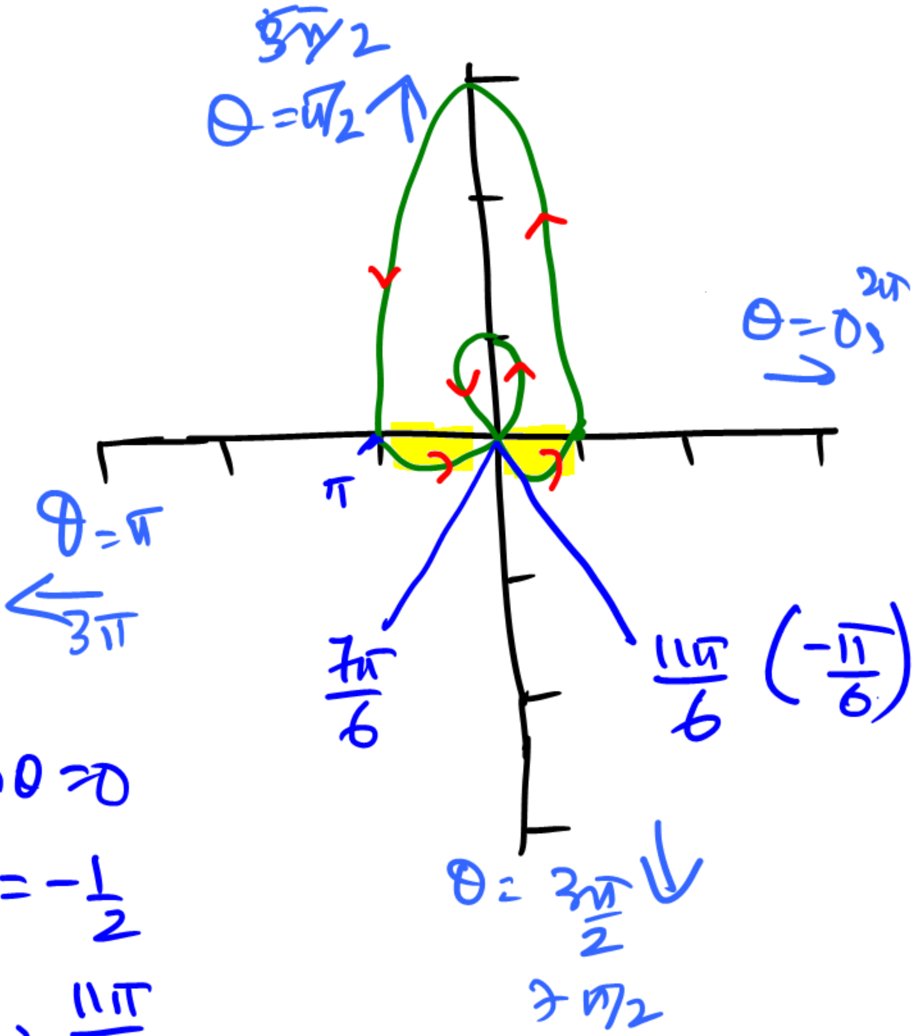
$\sin\frac{2\pi}{11} = \sin\frac{24\pi}{11} = \sin\frac{46\pi}{11} = \cdots$

$\sin x = 2\pi$

$\sin\frac{x}{11} = 22\pi$

(*) $r = 1 + 2\sin\theta$

θ	r
0	1
$\frac{\pi}{2}$	3
π	1
$\frac{3\pi}{2}$	-1
2π	0



$$r = 0 \Rightarrow 1 + 2\sin\theta = 0$$

$$\sin\theta = -\frac{1}{2}$$

$$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$A = \int \frac{1}{2} r^2 \, d\theta = \cancel{2}\cdot\int_{\pi}^{7\pi/6} \frac{1}{2}(1+2\sin\theta)^2 \, d\theta$$

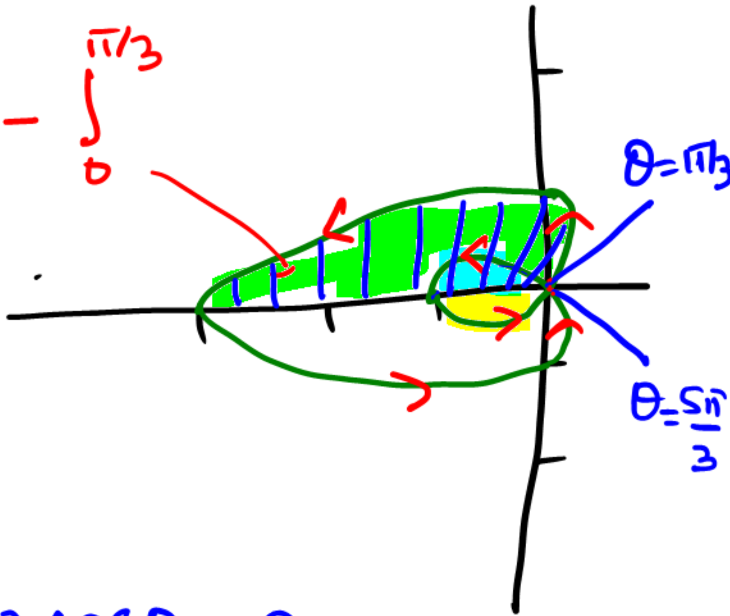
$$= \int_{11\pi/6}^{2\pi} 2\cdot\frac{1}{2}(1+2\sin\theta)^2 \, d\theta$$

$$= \int_{-\pi/6}^{0} 2\cdot\frac{1}{2}(1+2\sin\theta)^2 \, d\theta$$

(*) $r = 1 - 2\cos\theta$

θ	r
0	-1
$\frac{\pi}{2}$	1
π	3
$\frac{3\pi}{2}$	1
2π	-1

$\int_{\pi/3}^{\pi} - \int_{0}^{\pi/3}$



$r = 0$

$\Rightarrow 1 - 2\cos\theta = 0$

$\Rightarrow \cos\theta = \frac{1}{2}$

$\Rightarrow \theta = \frac{\pi}{3},$

$\int \frac{1}{2} r^2 \, d\theta$

$\boxed{\int_{5\pi/3}^{\pi/3} \quad \int_{\pi/3}^{5\pi/3}}$ ✗

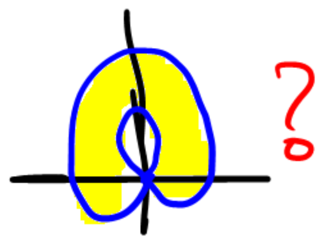
$\int_0^{\pi/3} 2 \cdot \frac{1}{2} r^2 \, d\theta$

$\int_{5\pi/3}^{2\pi} 2 \cdot \frac{1}{2} r^2 \, d\theta$

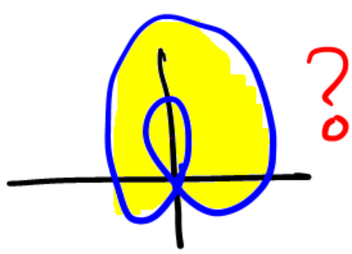
$\int_{5\pi/3}^{7\pi/3} \frac{1}{2} r^2 \, d\theta$

Inner Loop

Between Loop $\int_0^{2\pi} \frac{1}{2} r^2 \, d\theta - 2IL$



Outer Loop $\int_0^{2\pi} \frac{1}{2} r^2 \, d\theta - IL$



$\equiv$

(*) $r = \sin(3\theta)$

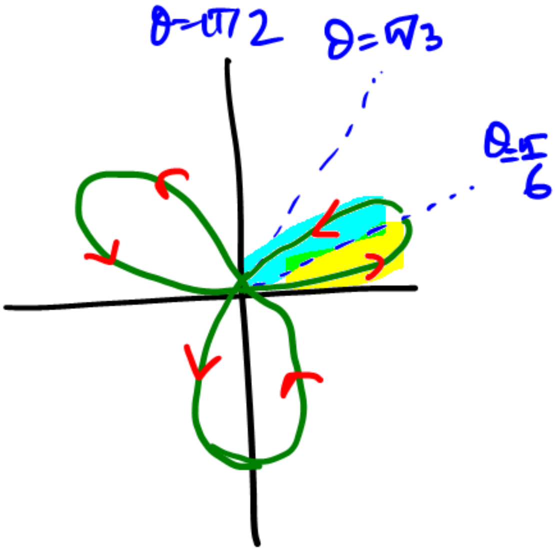
θ	r
0	0
$\frac{\pi}{6}$	1
$\frac{\pi}{3}$	0
$\frac{\pi}{2}$	-1
$\frac{2\pi}{3}$	0

$3\theta = \frac{\pi}{2}$

$\theta = \frac{\pi}{6}$

$3\theta = \pi$

$\theta = \frac{\pi}{3}$



Area of one petal

$$= \int_0^{\pi/3} \frac{1}{2} r^2 \, d\theta$$

$$= 2 \int_0^{\pi/6} \frac{1}{2} r^2 \, d\theta$$

$$= 2 \int_{\pi/6}^{\pi/3} \frac{1}{2} r^2 \, d\theta$$

Area of all petals

$$= 3 \int_0^{\pi/3} \frac{1}{2} r^2 \, d\theta$$

$$= 3.2 \int_0^{\pi/6} \frac{1}{2} r^2 \, d\theta$$