Lab 01/12

(*) $x(t) = 11 - \sin t \qquad y(t) = \cos t$

$\sin t = 11 - x$

$\sin^2 t + \cos^2 t = 1$

$(11-x)^2 + y^2 = 1$

$x^2 - 22x + 121 + y^2 = 1$

$x^2 - 22y + y^2 = -120$

$(-(x-11))^2 + y^2 = 1 \qquad 11 - x = -(x-11)$

$(-1)^2 (x-11)^2 + y^2 = 1$

$(x-11)^2 + y^2 = 1$

(*) $x(t) = 3 - 2\sin t \qquad y(t) = 5 + 3\cos t$

$\sin t = \frac{3-x}{2} \qquad \cos t = \frac{y-5}{3}$

$\sin^2 t + \cos^2 t = 1$

$\left(\frac{3-x}{2}\right)^2 + \left(\frac{y-5}{3}\right)^2 = 1$

$\left(\frac{-(x-3)}{2}\right)^2 + \left(\frac{y-5}{3}\right)^2 = 1$

$\left(\frac{x-3}{2}\right)^2 + \left(\frac{y-5}{3}\right)^2 = 1$

$x(t) = 3 - 2\sin t \qquad y(t) = 5 + 3\cos t$

Find equ^n of tangent line at $t = \frac{\pi}{3}$

$x(\frac{\pi}{3}) = 3 - 2\sin(\frac{\pi}{3}) = 3 - 2\frac{\sqrt{3}}{2} = 3 - \sqrt{3}$

$y(\frac{\pi}{3}) = 5 + 3\cos(\frac{\pi}{3}) = 3 + 3.\frac{1}{2} = \frac{9}{2}$

$(3 - \sqrt{3}, \frac{9}{2})$

$x'(t) = -2\cos t \qquad y'(t) = -3\sin t$

$x'(\frac{\pi}{3}) = -2.\frac{1}{2} \qquad y'(t) = -3.\frac{\sqrt{3}}{2}$

$= -1 \qquad = \frac{-3\sqrt{2}}{2}$

$$m = \left.\frac{y'(t)}{x'(t)}\right|_{t=\pi/3} = \frac{-\frac{3\sqrt{2}}{2}}{-1} = \frac{3\sqrt{2}}{2}$$

$y - \frac{9}{2} = \frac{3\sqrt{2}}{2}(x - (3 - \sqrt{3}))$

$2y - 9 = 3\sqrt{2}\,(x - (3 - \sqrt{3}))$

Horizontal asymp $\qquad t \in [0, 2\pi)$

$y'(t) = 0 \quad \& \quad x'(t) \neq 0$

$x'(t) = -2\cos t$

$-3\sin t = 0$

$\sin t = 0 \qquad t = 0 \quad x'(t) = -2 \neq 0$

$t = 0, \pi \qquad t = \pi \quad x'(t) = 2 \neq 0$

$x(0) = 3 - 2\sin 0 = 3 \qquad x(\pi) = 3 \qquad (3, 8)$

$y(0) = 5 + 3\cos 0 = 8 \qquad y(\pi) = 2 \qquad (3, 2)$

(*) $r = 20 - 10\sin\theta$ at $\theta = 0$

$$x = r\cos\theta = (20 - 10\sin\theta)\cos\theta$$
$$= 20\cos\theta - 10\sin\theta\cos\theta$$
$$= 20\cos\theta - 5(2\sin\theta\cos\theta)$$
$$= 20\cos\theta - 5\sin 2\theta$$

$$y = r\sin\theta = (20 - 10\sin\theta)\sin\theta$$
$$= 20\sin\theta - 10\sin^2\theta$$

$$x'(\theta) = -20\sin\theta - 10\cos\theta$$
$$y'(\theta) = 20\cos\theta - 20\sin\theta\cos\theta$$

$x'(0) = -10$ $\quad$ $y'(0) = 20$

$m = \frac{20}{-10} = -2$ $\quad$ $x(0) = 20$

$y(0) = 0$

$$y - 0 = -2(x - 20)$$
$$y = -2x + 40 \Rightarrow y + 2x = 40$$

(*) $r = 5\cos 2\theta \qquad \theta = \frac{\pi}{2}$

$x(\theta) = r\cos\theta = 5\cos 2\theta\cos\theta$

$y(\theta) = r\sin\theta = 5\cos 2\theta\sin\theta$

$x'(\theta) = -10\sin 2\theta\cos\theta - 5\cos 2\theta\sin\theta$

$y'(\theta) = -10\sin 2\theta\sin\theta + 5\cos 2\theta\cos\theta$

$x'(\frac{\pi}{2}) = -5(-1)(1) = 5$

$y'(\frac{\pi}{2}) = 5(-1)(0) = 0$

$m = \frac{y'}{x'} = 0$

$x(\frac{\pi}{2}) = 0 \qquad y(\frac{\pi}{2}) = 5(-1)\cdot 1 = -5$

$y - (-5) = 0(x-0) \Rightarrow y = -5$

(*) $x(t) = 2\cos t \qquad y(t) = 2\sin(2t)$

$x'(t) = -2\sin t \qquad y'(t) = -4\cos 2t$

$y'(t) = 0 \Rightarrow \cos 2t = 0 \qquad \cos x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$2t = \frac{\pi}{2}, \frac{3\pi}{2}$

$t = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$t = \frac{\pi}{4} \qquad x(\frac{\pi}{4}) = 2\cdot\frac{\sqrt{2}}{2} = \sqrt{2} \qquad y(\frac{\pi}{4}) = 2\sin(\frac{\pi}{2}) = 2$

$t = \frac{3\pi}{4} \qquad x(\frac{3\pi}{4}) = -\sqrt{2} \qquad y(\frac{3\pi}{4}) = 2\sin(\frac{3\pi}{2}) = -2$

$t = \frac{5\pi}{4} \qquad x(\frac{5\pi}{4}) = -\sqrt{2} \qquad y(\frac{5\pi}{4}) = 2$

$t = \frac{7\pi}{4} \qquad x(\frac{7\pi}{4}) = \sqrt{2} \qquad y(\frac{7\pi}{4}) = -2$

(*) $x(t) = e^t$

$y(t) = 8 - e^{2t}$

$y(t) = 8 - (e^t)^2$

$y = 8 - x^2$

(*) $x(t) = \cos^3 2t \qquad y(t) = \sin^3 2t \qquad t = \frac{\pi}{3}$

$x'(t) = -3\cos^2 2t \cdot 2\sin 2t \qquad y'(t) = 3\sin^2 2t \cdot 2\cos 2t$

$x'(\frac{\pi}{3}) = -6\left(\cos\left(\frac{2\pi}{3}\right)\right)^2 \sin\left(\frac{2\pi}{3}\right) \qquad y'(\frac{\pi}{3}) = 6\left(\sin\left(\frac{2\pi}{3}\right)\right)^2 \cos\left(\frac{2\pi}{3}\right)$

$= -6\left(-\frac{1}{2}\right)^2 \frac{\sqrt{3}}{2} \qquad = 6\left(\frac{\sqrt{3}}{2}\right)^2\left(-\frac{1}{2}\right)$

$= -6\,\frac{1}{4}\frac{\sqrt{3}}{2} = \frac{-3\sqrt{3}}{4} \qquad = \cancel{6}^{3}\,\frac{3}{4}\,\frac{1}{\cancel{2}} = -\frac{9}{4}$

$m = \frac{-9/\cancel{4}}{-3\sqrt{3}/\cancel{4}} = \frac{3}{\sqrt{3}} = \sqrt{3}$

(*) $r = 4\sin 2\theta \qquad r = 4\sin\theta$

$4\sin 2\theta = 4\sin\theta$

$\sin 2\theta - \sin\theta = 0$

$2\sin\theta\cos\theta - \sin\theta = 0$

$\sin\theta\,(2\cos\theta - 1) = 0$

$\sin\theta = 0 \qquad 2\cos\theta - 1 = 0$

$\theta = 0, \pi, 2\pi, \ldots \qquad \cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

$(0,0)$

$\theta = \frac{\pi}{3}$

$r = 4\cdot\frac{\sqrt{3}}{2} = 2\sqrt{3}$

$\theta = \frac{5\pi}{3}$

$r = 4\left(-\frac{\sqrt{3}}{2}\right)$

$= -2\sqrt{3}$

$x = r\cos\theta \qquad \theta = \frac{\pi}{3}$

$= 4\sin\theta\cos\theta$

$= 4\sin\frac{\pi}{3}\cos\frac{\pi}{3})$

$= 4\frac{\sqrt{3}}{2}\cdot\frac{1}{2} = \sqrt{3}$

$y = 4\sin^2\theta$

$= 4\left(\frac{\sqrt{3}}{2}\right)^2 = \sqrt{3}$

$(\sqrt{3}, \sqrt{3})$

(*) $x = x(t) \qquad y = y(t)$

$f(x) = x^8 + 5x^2 - 5 \qquad (2,-5)\ (4,5)$

$y = f(x)$

$y(t) = f(t)$

$x(t) = t \qquad y(t) = t^8 + 5t^2 - 5$

$t \in [2,4]$

(*) $r = 16 \qquad \theta = \dfrac{\pi}{2} \qquad r = 8\sec\theta$

$r\cos\theta = 8$

$x = 8$

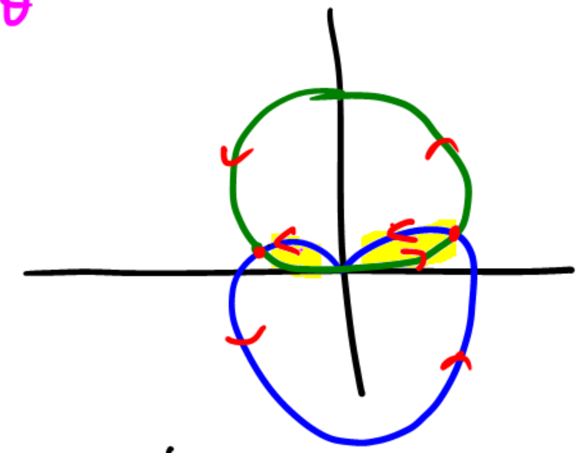
$16 = 8\sec\theta$

$\cos\theta = \dfrac{1}{2}$

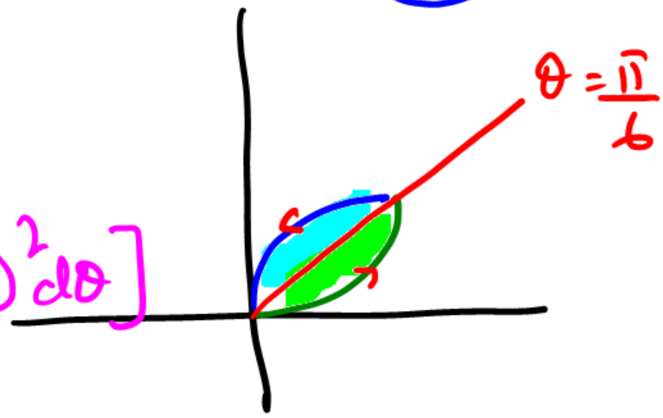
$\theta = \pi/3, \dfrac{5\pi}{3}$

$$2\left[\int_{\pi/3}^{\pi/2} \frac{1}{2}(16)^2\, d\theta + \int_{0}^{\pi/3} \frac{1}{2}(8\sec\theta)^2\, d\theta\right]$$

(*) $r = 3 - 3\sin\theta \qquad r = 3\sin\theta$



$$3 - 3\sin\theta = 3\sin\theta$$
$$3 = 6\sin\theta$$
$$\sin\theta = \frac{1}{2}$$
$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$



$$2\left[\frac{1}{2}\int_0^{\pi/6}(3\sin\theta)^2 d\theta + \frac{1}{2}\int_{\pi/6}^{\pi/2}(3-3\sin\theta)^2 d\theta\right]$$

(*) $x(t) = t^2 \quad y(t) = t^3 \qquad t\ [0,3]$

$(x(t), y(t)) = (t^2, t^3)$

$v(t) = (2t, 3t^2) \qquad \|v(t)\| = \sqrt{(2t)^2 + (3t^2)^2}$

$= \sqrt{4t^2 + 9t^4}$

Inti vel $t=0 \quad \|v(0)\| = \sqrt{4\cdot 0 + 9\cdot 0} = 0$

Term vel $t=3 \quad \|v(3)\| = \sqrt{4\cdot 9 + 9\cdot 81} = \sqrt{765}$

$$\int_0^3 \sqrt{4t^2+9t^4}\,dt = \int_0^3 \sqrt{t^2(4+9t^2)}\,dt = \int_0^3 t\sqrt{4+9t^2}\,dt$$

$u = 4 + 9t^2$

$du = 18t\,dt$

$$\int t\sqrt{4+9t^2}\,dt = \frac{1}{18}\int\sqrt{u}\,du$$
$$= \frac{1}{18}\frac{u^{3/2}}{3/2} + C = \frac{(4+9t^2)^{3/2}}{27} + C$$

(*) $x(t) = 3e^t \sin t \qquad y(t) = 3e^t \cos t$

$x'(t) = 3e^t \sin t + 3e^t \cos t = 3e^t(\sin t + \cos t)$

$y'(t) = 3e^t \cos t - 3e^t \sin t = 3e^t(\cos t - \sin t)$

$$\|v(t)\| = \sqrt{(x'(t))^2 + (y'(t))^2}$$

$$= \sqrt{9e^{2t}(\sin^2 t + \cos^2 t + 2\sin t\cos t) + 9e^{2t}(\cos^2 t + \sin^2 t - 2\sin t\cos t)}$$

$$= \sqrt{9e^{2t}(2(\sin^2 t + \cos^2 t))}$$

$= 3e^t\sqrt{2} \qquad t \in [0, \pi]$

Initial $= 3\sqrt{2} \qquad$ Final $= 3e^{\pi}\sqrt{2}$

Dis $= \int_0^{\pi} 3\sqrt{2}e^t\,dt = 3\sqrt{2}e^t\Big|_0^{\pi} = 3\sqrt{2}(e^{\pi} - 1)$

(*) $x(t) = 11\cos t \qquad y(t) = 11\sin 2t \qquad$ Hor tan.

$y'(t) = 22\cos 2t$

$y'(t) = 0 \Rightarrow \cos 2t = 0$

$\Rightarrow 2t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$

$t = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

(*) $x = x(t)$ $y = y(t)$ $t \in [0, 1]$ $(-8, -2)$ (x_0, y_0)

$(5, 8)$ (x_1, y_1)

$$x(t) = -8 + t(5 - (-8))$$

$$y(t) = -2 + t(8 - (-2))$$

$x(t) = -8 + 13t$ $y(t) = -2 + 10t$

line segment $t \in [0, 1]$

line $t \in (-\infty, \infty)$