Lab 01/13

$\circledast$ Half life 1620   How long reduce by 85%

$$A = A_0 e^{kt}$$

$$\frac{A_0}{2} = A_0 e^{1620k}$$

$$\ln\left(\frac{1}{2}\right) = 1620k$$

$$k = \frac{\ln(1/2)}{1620}$$

A_0

$A = 0.15 A_0$

$$A = A_0 e^{kt}$$

$$0.15\cancel{A_0} = \cancel{A_0} e^{kt}$$

$$\ln(0.15) = kt$$

$$t = \frac{\ln(0.15)}{k} = \frac{1620 \ln(0.15)}{\ln(0.5)}$$

$\circledast$

$$y = -3x^2 + 2$$

$$y' = -6x$$

$$-4y' + 12x^2$$

$$= 24x + 12x \neq 0$$

$$-4y' + 12x^2 = 0$$

$$-4\frac{dy}{dx} + 12x^2 = 0$$

$$-4dy = -12x^2 dx$$

$$dy = 3x^2 dx$$

$$y = x^3 + c$$

(*) $\int \frac{4x^2}{\sqrt{10+x^2}}\,dx$ $\qquad x=\sqrt{10}\tan\theta$

$\qquad dx=\sqrt{10}\sec^2\theta\,d\theta$

$$= \int \frac{4\cdot 10\tan^2\theta\,\cancel{\sqrt{10}}\sec^{\cancel{2}}\theta\,d\theta}{\cancel{\sqrt{10}}\,\cancel{\sec\theta}}$$

$$= 40\int \tan^2\theta\sec\theta\,d\theta$$

$$= 40\int (\sec^2\theta-1)\sec\theta\,d\theta$$

$$= 40\left[\int \sec^3\theta\,d\theta - \int\sec\theta\,d\theta\right]$$

$$= 40\left[\frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\sec\theta+\tan\theta| - \ln|\sec\theta+\tan\theta|\right] + C$$

$$= 40\cdot\frac{1}{2}\left[\sec\theta\tan\theta - \ln|\sec\theta+\tan\theta|\right] + C$$

$$= 20\left[\frac{\sqrt{x^2+10}}{\sqrt{10}}\,\frac{x}{\sqrt{10}} - \ln\left|\frac{\sqrt{x^2+10}}{\sqrt{10}}+\frac{x}{\sqrt{10}}\right|\right] + C$$

$$= 2x\sqrt{x^2+10} - 20\ln\left|\frac{\sqrt{x^2+10}+x}{\sqrt{10}}\right| + C$$

(*) $\int x^3 \sin(11x^2)\,dx$

$z = 11x^2 \quad x^2 = \frac{z}{11}$

$dz = 22x\,dz$

$= \int x \cdot x^2 \sin(11x^2)\,dx$

$= \frac{1}{22}\int \frac{z}{11} \sin z\,dz$

$u = z \quad dv = \sin z\,dz$

$du = dz \quad v = -\cos z$

$= \frac{1}{22 \cdot 11}\left[-z\cos z + \int \cos z\,dz\right]$

$= \frac{1}{22 \cdot 11} z\cos z + \frac{\sin z}{22 \cdot 11} + c$

$= -\frac{1}{22 \cdot \cancel{11}}\cancel{11}x^2 \cos 11x^2 + \frac{\sin(11x^2)}{22 \cdot 11} + c$

$= -\frac{x^2}{22}\cos(11x^2) + \frac{\sin(11x^2)}{242} + c$

(*) $\sum \frac{3k-1}{\sqrt{k^5+2}} = \sum a_k$

$\sum b_k = \sum \frac{1}{k^{5/2-1}}$

$= \sum \frac{1}{k^{3/2}}$

$\lim_{k\to\infty} \frac{a_k}{b_k} = \lim_{k\to\infty} \frac{3k^{5/2} - k^{3/2}}{\sqrt{k^5+2}} = 3$

$\sum b_k$ is convgt by p-series test as $p = \frac{3}{2} > 1$

$\therefore \sum a_k$ is convgt by LCT

// $\sum \frac{4k+1}{\sqrt{k^4+1}} = \sum a_k$

$\sum b_k = \sum \frac{1}{k}$

$\circledast$ $\sum_{0}^{\infty} \frac{3^{k+1}}{4^k} = \sum_{0}^{\infty} \left(\frac{3}{4}\right)^k \cdot 3$

$r = \frac{3}{4}$

$|r| = \frac{3}{4} < 1$

$\frac{a}{1-r}$

$= 3 \frac{1}{1 - \frac{3}{4}}$

$= \frac{3}{1/4} = 12$

$\sum_{1}^{\infty} (-1)^k \frac{3^{3k+1}}{7^{2k-1}} = \sum_{1}^{\infty} \frac{(-1)^k \, 27^k \cdot 3}{49^k \cdot 7^{-1}}$

$r = -\frac{27}{49}$

$|r| = \frac{27}{49} < 1$

$= 21 \sum_{1}^{\infty} \left(-\frac{27}{49}\right)^k$

$= 21 \frac{-27/49}{1 - \left(-\frac{27}{49}\right)}$

$= 21 \frac{-27/49}{1 + \frac{27}{49}} = 21 \frac{-27/\cancel{49}}{76/\cancel{49}}$

$= -\frac{21 \cdot 27}{76}$

$\circledast$ $x(t) = \cos^3(7t)$ $\quad y(t) = \sin^3(7t)$ $\quad t = \frac{\pi}{6}$

$\frac{dx}{dt} = 3\cos^2(7t)(-\sin 7t) \cdot 7$

$x'\left(\frac{\pi}{6}\right) = 21 \cos^2\left(\frac{7\pi}{6}\right)\left(-\sin\left(\frac{7\pi}{6}\right)\right)$

$= 21 \left(-\frac{\sqrt{3}}{2}\right)^2 \left(-\left(-\frac{1}{2}\right)\right) = 21 \, \frac{3}{4} \cdot \frac{1}{2} = \frac{63}{8}$

$$y = \sin^3(7t)$$

$$y' = 3\sin^2 7t\,(\cos 7t)\,7.$$

$$y'\left(\frac{\pi}{6}\right) = 3\left(-\frac{1}{2}\right)^2\left(-\frac{\sqrt{3}}{2}\right)\cdot 7$$

$$= -\frac{21\sqrt{3}}{8}$$

$$m = \frac{\frac{-21\sqrt{3}}{\cancel{8}}}{\frac{63}{\cancel{8}}} = \frac{-\cancel{21}\sqrt{3}}{\cancel{63}\,3} = \frac{-\sqrt{3}}{3}$$

(*) $$|R_n(x)| \le \frac{M}{(n+1)!}|x|^{n+1}$$ $\quad |f^{(n+1)}(c)| \le M.$

$f(5)\quad 0.001$

$$\frac{1}{(n+1)!}(5)^{n+1} \le 0.001$$ $\quad |f^{n}(x)| \le 1.$

$$\frac{5^{n+1}}{(n+1)!} \le \frac{1}{1000}$$

$n = 14$ $$\frac{5^{14+1}}{(14+1)!} = 0.0233$$

$n=15 \quad 0.007$

$n=16 \quad 0.002$

$n=17 \quad 0.0005 \leftarrow \boxed{n=17}$

(2) $$\frac{|x|^{n+1}}{(n+1)!}M \le 0.001$$ $$\frac{1}{(n+1)!} \le 0.001 \quad \frac{1}{1000}$$

$n=5 \quad \frac{1}{6!} \quad \frac{1}{720}$

$\underline{\underline{n=6}} \quad \frac{1}{7!} \quad \frac{1}{5040}$

$f(x) = \cos x / \sin x$ $\quad M = 1$

$|f^n(x)| \le M$

$f(x) = e^{-x}$ $\quad [0, 5]$

$f'(x) = -e^{-x}$

$f''(x) = e^{-x}$ $\quad f^n(x) = (-1)^n e^{-x}$

$= (-1)^n$

$\cancel{e^{-x}} \rightarrow$

$M = 1$

$$\sum \frac{2^k x^{2k}}{(k+1)^2}$$

$$\left|\frac{a_{k+1}}{a_k}\right| = \left|\frac{2^{k+1} x^{2k+2}}{(k+2)^2} \cdot \frac{(k+1)^2}{2^k x^{2k}}\right|$$

$$= 2x^2\left(\frac{k+1}{k+2}\right)^2 \xrightarrow[k\to\infty]{} 2x^2$$

$2x^2 < 1$

$x^2 < \frac{1}{2} \Rightarrow -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$

$$x = \frac{1}{\sqrt{2}} \quad \sum \frac{2^k \left(\frac{1}{\sqrt{2}}\right)^{2k}}{(k+1)^2} = \sum \frac{\cancel{2^k}\frac{1}{\cancel{2^k}}}{(k+1)^2} = \sum \frac{1}{(k+1)^2}$$

$\sum \frac{1}{(k+1)^2} \le \sum \frac{1}{k^2} \rightarrow$ conv by p-series test

$\hookrightarrow$ conv by by DCT

$x = -\frac{1}{\sqrt{2}}$ $\quad \sum \frac{2^k \left(-\frac{1}{\sqrt{2}}\right)^{2k}}{(k+1)^2} = \sum \frac{2^k \frac{1}{2^k}}{(k+1)^2}$

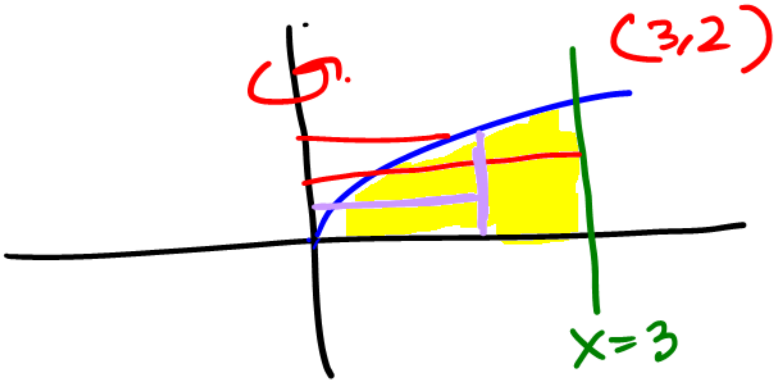
$$= \sum \frac{1}{(k+1)^2}$$

conv. same as before

$$\left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right] \quad \left[-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right]$$

(*) $x = \frac{3y^3}{8}$ $\quad x = 3$

$y = 0$



$\frac{3y^3}{8} = 3 \Rightarrow y^3 = 8$

$\Rightarrow y = 2$

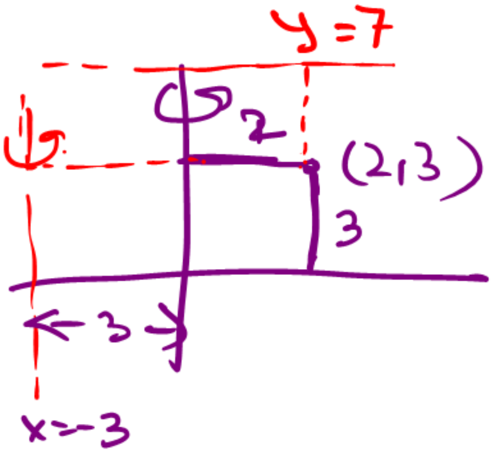
D/W $\quad \pi \int_0^2 (3)^2 - \left(\frac{3y^3}{8}\right)^2 dy$

shell $\quad 2\pi \int_0^3 x \left(\frac{8}{3}x\right)^{1/3} dx$

(*) $f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \frac{f^4(0)x^4}{4!}$

$-2 + 2x + \frac{3}{2}x^2 + \frac{4}{6}x^3 + \frac{1}{24}x^4$

(*) $A = 10 \quad C(2,3)$



$V = 2\pi R A$

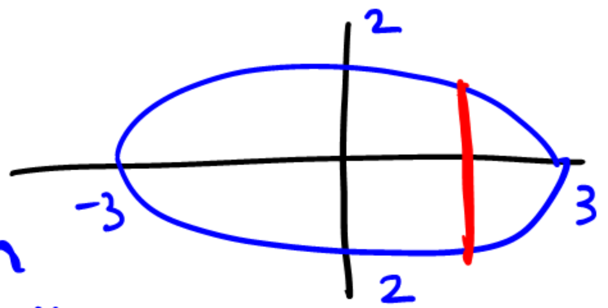
<u>y-axis</u> $V = 2.\pi.2.10 = 40\pi$

<u>x-axis</u> $V = 2.\pi.3.10 = 60\pi$

<u>$x=-3$</u> $V = 2\pi(3+2).10 = 100\pi$

<u>$y=6$</u> $V = 2\pi(7-3)\cdot 10 = 80\pi$

(*) $\frac{x^2}{9} + \frac{y^2}{4} = 1$



Find the volume of region formed by cross-section $\parallel$'lr y-axis $\perp$ x-axis and are square

$$\frac{y^2}{4} = 1 - \frac{x^2}{9} \Rightarrow y^2 = \frac{36-4x^2}{9}$$

$$y = \frac{\pm\sqrt{36-4x^2}}{3}$$

$$\text{side} = \frac{2\sqrt{36-4x^2}}{3}$$

$$\text{Area} = (\text{side})^2 = \frac{4}{9}(36-4x^2)$$

$$V = 2\int_0^3 \frac{4}{9}(36-4x^2)\,dx$$

$$\left(\frac{n}{n-6}\right)^{-2n}$$

$$= \left(\frac{n-6}{n}\right)^{2n} = \left(1-\frac{6}{n}\right)^{2n} \xrightarrow[n\to\infty]{} e^{-12}$$

$r = 1 - 2\cos\theta$

θ	
0	-1
$\frac{\pi}{2}$	1
π	3
$\frac{3\pi}{2}$	1
2π	-1

$r = 0$

$1 - 2\cos\theta = 0$

$\cos\theta = \frac{1}{2}$

$\theta = \pi/3, 5\pi/3$

$r = \sin(3\theta)$

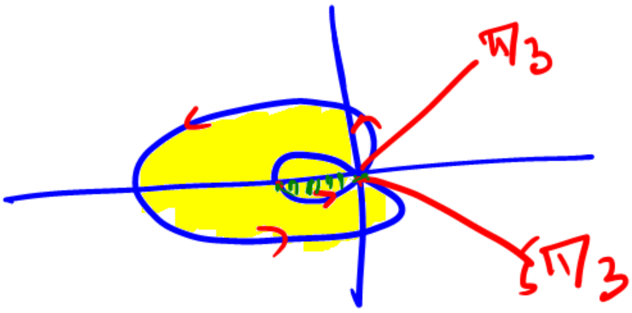
$3\theta = \frac{\pi}{2}$

$3\theta = \pi$

θ	
0	
$\frac{\pi}{6}$	
$\frac{\pi}{3}$	
$\frac{\pi}{2}$	

$$I.L = 2\int_0^{\pi/3} \frac{1}{2} r^2 \, d\theta$$

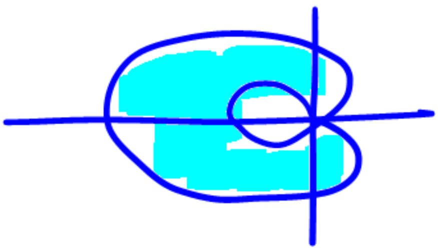
$$= 2\int_{5\pi/3}^{2\pi} \frac{1}{2} r^2 \, d\theta$$



$$\int_0^{2\pi} - IL$$

OR $2\int_{\pi/3}^{\pi}$

OR $\int_{\pi/3}^{5\pi/3}$



$$\int_0^{2\pi} - 2 \cdot IL$$

OR $2\left[\int_{\pi/3}^{\pi} - \int_0^{\pi/3}\right]$