Lab 01/14

$$\int x^3\cos(10x^2)\,dx$$

$t = 10x^2 \quad x^2 = \frac{t}{10}$

$dt = 20x\,dx$

$$= \int x\,x^2\cos(10x^2)\,dx$$

$$= \frac{1}{20}\frac{1}{10}\int t\cos(t)\,dt$$

$u = t \quad dv = \cos t\,dt$

$du = dt \quad v = \sin t$

$$= \frac{1}{200}\left[t\sin t - \int \sin t\,dt\right]$$

$$= \frac{1}{200}\left[t\sin t + \cos t\right] + c$$

$$= \frac{\cancel{10}x^2\sin(10x^2)}{20\cancel{0}} + \frac{\cos(10x^2)}{200} + c$$

$$= \frac{x^2\sin(10x^2)}{20} + \frac{\cos(10x^2)}{200} + c$$

$$\int x\ln(1+x^2)\,dx$$

$u = \ln(1+x^2) \quad dv = x\,dx$

$du = \frac{2x}{1+x^2}dx \quad v = \frac{x^2}{2}$

$$= \frac{x^2}{2}\ln(1+x^2) - \int \frac{x^3}{1+x^2}\,dx$$

$$= \frac{x^2}{2}\ln(1+x^2) - \int \frac{x^3+x-x}{1+x^2}\,dx$$

$$= \frac{x^2}{2}\ln(1+x^2) - \int\left(\frac{x^3+x}{1+x^2} - \frac{x}{1+x^2}\right)dx$$

$$= \frac{x^2}{2}\ln(1+x^2) - \int \frac{x\cancel{(1+x^2)}}{\cancel{1+x^2}}\,dx + \int \frac{x}{1+x^2}\,dx$$

$$= \frac{x^2}{2}\ln(1+x^2) - \frac{x^2}{2} + \frac{1}{2}\ln|1+x^2| + c$$

$$\begin{array}{r|l} & x \\ x^2+1 & x^3 \\ & x^3 + x \\ \hline & -x \end{array}$$

$$x - \frac{x}{1+x^2}$$

* $\int_0^2 \frac{1}{\sqrt{x^3+4}}\,dx \qquad f(x) = \frac{1}{\sqrt{x^3+4}}$

$n=4 \quad x_0 = 0 \quad x_1 = 0.5 \quad x_2 = 1 \quad x_3 = 1.5 \quad x_4 = 2$

$T = \frac{b-a}{2n}\left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)\right]$

* $\sum \left|\frac{(-1)^k (k+3)}{k^2+4k+1}\right| = \sum \frac{k+3}{k^2+4k+1} = \sum a_k \qquad \sum b_k = \sum \frac{1}{k}$

$\frac{a_k}{b_k} = \frac{k^2+3k}{k^2+4k+1} \to 1$ as $k \to \infty$

$\sum b_k$ is div by p-series test as $p=1$

$\sum a_k$ is div by LCT

Hence the series is not abs conv.

$a_k = \frac{k+3}{k^2+4k+1}$ (i) mon dec

(ii) $\lim_{k\to\infty} a_k = 0$

so by alt series test the series in convgt

Hence the series is C.C.

* $f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \ldots$

$f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \frac{f^4(0)}{4!}x^4$

$1 - 2x + \frac{1}{2}x^2 - \frac{\cancel{3}}{2\cancel{6}}x^3 - \frac{\cancel{2}}{12\cancel{24}}x^4$

(*) $|f^n(x)| \le 1$ for all n

$\varepsilon = 0.01$

$f(3)$

$$|R_n(x)| \le \frac{M}{(n+1)!} |x|^{n+1}$$

$$= \frac{1}{(n+1)!} (3)^{n+1}$$

$|f^n(x)| \le M$

$M = 1$

$P_n(3)$

$f(3)$

$$\frac{3^{n+1}}{(n+1)!} \le 0.01$$

$n = 8$ 0.054

$n = 9$ 0.016

$n = 10$ 0.0044 ✓

$n = 10$

$f(1)$

$$\frac{1}{(n+1)!} \le \frac{1}{100}$$

$n = 3$ $\frac{1}{4!} = \frac{1}{24}$

$n = 4$ $\frac{1}{5!} = \frac{1}{120}$

$n = 5$

$\circledast$ $f(x) = \ln(1+x)$ $\quad P_n(0.5)$

$x = 0.5$

$$\frac{\max\{f^{n+1}(c)\}(0.5)^{n+1}}{(n+1)!} \leq 0.001$$

$|f^n(x)| \leq M$

$f(x) = \ln(1+x)$

$|f'(x)| = \left|\frac{1}{1+x}\right| < 1 = 0!$ $\quad (1-1)!$

$|f''(x)| = \left|-\frac{1}{(1+x)^2}\right| < 1 = 1!$ $\quad (2-1)!$

$|f'''(x)| = \left|\frac{2}{(1+x)^3}\right| < 2 = 2!$ $\quad (3-1)!$

$|f^4(x)| = \left|\frac{-6}{(1+x)^4}\right| < 6 = 3!$ $\quad (4-1)!$

$|f^n(x)| \leq (n-1)!$

$|f^{n+1}(c)| \leq n!$

$$\frac{n!}{(n+1)!}(0.5)^{n+1} \leq 0.001$$

$$\frac{1}{n+1}(0.5)^{n+1} \leq 0.001$$

(*) $\sum \frac{5ke^{-k^2}}{4} = \sum \frac{5k}{4e^{k^2}}$

$(a_k)^{1/k} = \left(\frac{k}{e^{k^2}}\right)^{1/k} = \frac{k^{1/k}}{e^k} \longrightarrow 0 \text{ as } k\to\infty$

< 1

conv by Root test

or

$\frac{a_{k+1}}{a_k} = \frac{k+1}{e^{(k+1)^2}} \cdot \frac{e^{k^2}}{k} = \left(\frac{k+1}{k}\right) e^{k^2-(k+1)^2}$

$= \left(\frac{k+1}{k}\right) e^{\cancel{k^2} - \cancel{k^2} - 1 - 2k}$

$= \left(\frac{k+1}{k}\right) \frac{1}{e^{2k+1}} \to 0 < 1$ as $k\to\infty$

Conv by Ratio test

$a_n = ne^{-n^2}$

Glb 0

LUB $\frac{1}{e}$

Monotonicity

$\frac{1}{e}$ $\frac{2}{e^2}$ $\frac{3}{e^3}$ ...

Mono Dec

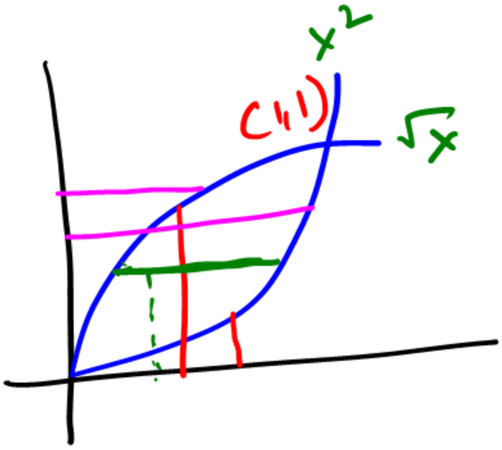
$\frac{a_{n+1}}{a_n} = \frac{n+1}{e^{(n+1)^2}} \cdot \frac{e^{n^2}}{n} = \left(\frac{n+1}{n}\right)\frac{1}{e^{2n+1}} < 1$

$a_{n+1} < a_n$

or

$a_n' = e^{-n^2} - 2n^2e^{-n^2} = e^{-n^2}(1-2n^2) < 0$

(*) $f(x) = x^2$ $\quad g(x) = \sqrt{x}$



x-axis

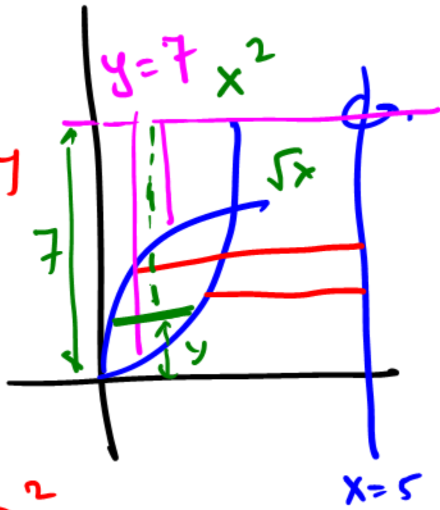
D/W $\pi \int_0^1 (\sqrt{x})^2 - (x^2)^2 \, dx$

Shell $2\pi \int_0^1 y(\sqrt{y} - y^2)\, dy$

y-axis

D/W $\pi \int_0^1 (\sqrt{y})^2 - (y^2)^2 \, dy$

Shell $2\pi \int_0^1 x\,(\sqrt{x} - x^2)\, dx$



x = 5

D/W $\pi \int_0^1 (5 - y^2)^2 - (5 - \sqrt{y})^2 \, dy$

shell $2\pi \int_0^1 (5 - x)(\sqrt{x} - x^2)\, dx$

y = 7

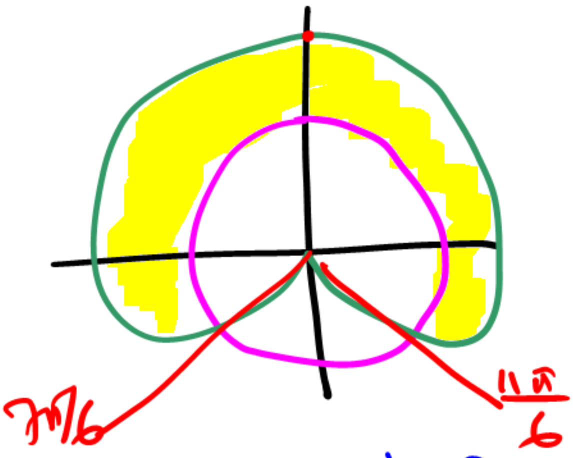
D/W $\pi \int_0^1 (7 - x^2)^2 - (7 - \sqrt{x})^2 \, dx$

shell $2\pi \int_0^1 (7 - y)(\sqrt{y} - y^2)\, dy$

(*) $r = 2$

$r = 4 + 4\sin\theta$

$$\int_{-\pi/6}^{7\pi/6} \frac{1}{2}\left[(4+4\sin\theta)^2 - (2)^2\right] d\theta$$



$2 = 4 + 4\sin\theta$

$-\frac{1}{2} = \sin\theta$

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

$$2\int_{-\pi/6}^{\pi/2} \left(\qquad \right)$$

$$2\int_{\pi/2}^{7\pi/6} \left(\qquad \right)$$

(*) $\left(\frac{n}{n-3}\right)^{-5n} = \left(\frac{n-3}{n}\right)^{5n}$

$= \left(1 - \frac{3}{n}\right)^{5n} \longrightarrow e^{-15}$

$\left(1 + \frac{a}{n}\right)^{bn} \longrightarrow e^{ab}$

(*) $\sum \frac{n^2 (x-3)^{n+2}}{5^n (n+1)!}$ $\quad f'(5) = \sum \frac{n^2\, 2^{n+2}}{5^n (n+1)!}$

$\sum \frac{n^2 (n+2)(x-3)^{n+1}}{5^n (n+1)!}$

⊛ $\sum \frac{3^n (x-3)^{n+1}}{(n+1)!}$ $F(3)=0$

$\int \sum \frac{3^n (x-3)^{n+1}}{(n+1)!}$

$= \sum \frac{3^n (x-3)^{n+2}}{(n+2)(n+1)!} + C$

$= \sum \frac{3^n (x-3)^{n+2}}{(n+2)!} + C$

$F(3) = 0 \Rightarrow C = 0$