Dialog box:

Stat ➤ Nonparametrics ➤ 1-Sample Wilcoxon

Type *C1* in **Variables.** Choose **Test median.** Type 5.05 in the text box. Click **OK.**

Session command:

```
MTB > WTEST 5.05 C1;
SUBC>   Alternative 0.
```

Output:

Wilcoxon Signed Rank Test: C1

```
TEST OF MEDIAN = 5.050 VERSUS MEDIAN N.E. 5.050

                  N FOR  WILCOXON            ESTIMATED
              N   TEST  STATISTIC P-VALUE      MEDIAN
C1           15     15       86.0   0.148       5.747
```

FIGURE 13.4.1 MINITAB procedure and output for Example 13.4.1.

Wilcoxon Matched-Pairs Signed-Ranks Test The Wilcoxon test may be used with paired data under circumstances in which it is not appropriate to use the paired-comparisons t test described in Chapter 7. In such cases obtain each of the n d_i values, the difference between each of the n pairs of measurements. If we let $\mu_D =$ the mean of a population of such differences, we may follow the procedure described above to test any one of the following null hypotheses: $H_0 : \mu_D = 0$, $H_0 : \mu_D \geq 0$, and $H_0 : \mu_D \leq 0$.

Computer Analysis Many statistics software packages will perform the Wilcoxon signed-rank test. If, for example, the data of Example 13.4.1 are stored in Column 1, we could use MINITAB to perform the test as shown in Figure 13.4.1.

EXERCISES

13.4.1 Sixteen laboratory animals were fed a special diet from birth through age 12 weeks. Their weight gains (in grams) were as follows:

63	68	79	65	64	63	65	64	76	74	66	66	67	73	69	76

Can we conclude from these data that the diet results in a mean weight gain of less than 70 grams? Let $\alpha = .05$, and find the p value.

13.4.2 Amateur and professional singers were the subjects of a study by Grape et al. (A-2). The researchers investigated the possible beneficial effects of singing on well-being during a single singing lesson. One of the variables of interest was the change in cortisol as a result of the signing lesson. Use the data in the following table to determine if, in general, cortisol (nmol/L) increases after a singing lesson. Let $\alpha = .05$. Find the p value.

Subject	1	2	3	4	5	6	7	8
Before	214	362	202	158	403	219	307	331
After	232	276	224	412	562	203	340	313

Source: Data provided courtesy of Christina Grape, M.P.H., Licensed Nurse.

13.4.3 In a study by Zuckerman and Heneghan (A-3), hemodynamic stresses were measured on subjects undergoing laparoscopic cholecystectomy. An outcome variable of interest was the ventricular end diastolic volume (LVEDV) measured in milliliters. A portion of the data appear in the following table. Baseline refers to a measurement taken 5 minutes after induction of anesthesia, and the term "5 minutes" refers to a measurement taken 5 minutes after baseline.

	LVEDV (ml)	
Subject	Baseline	5 Minutes
1	51.7	49.3
2	79.0	72.0
3	78.7	87.3
4	80.3	88.3
5	72.0	103.3
6	85.0	94.0
7	69.7	94.7
8	71.3	46.3
9	55.7	71.7
10	56.3	72.3

Source: Data provided courtesy of R. S. Zuckerman, MD.

May we conclude, on the basis of these data, that among subjects undergoing laparoscopic cholecystectomy, the average LVEDV levels change? Let $\alpha = .01$.

13.5 THE MEDIAN TEST

A nonparametric procedure that may be used to test the null hypothesis that two independent samples have been drawn from populations with equal medians is the median test. The test, attributed mainly to Mood (2) and Westenberg (3), is also discussed by Brown and Mood (4).

We illustrate the procedure by means of an example.

EXAMPLE 13.5.1

Do urban and rural male junior high school students differ with respect to their level of mental health?

Solution:

1. **Data.** Members of a random sample of 12 male students from a rural junior high school and an independent random sample of 16 male

Dialog box: **Session command:**

Stat ➤ Nonparametrics ➤ Mood's Median Test MTB > Mood C1 C2.

Type *C1* in **Response** and *C2* in **Factor.** Click **OK.**

Output:

Mood Median Test: C1 versus C2

```
Mood median test of C1

Chisquare = 2.33 df = 1 p = 0.127

                                          Individual 95.0% CIs
   C2  N<=  N>  Median   Q3-Q1  --------+---------+---------+--------
    1   10   6    27.0    15.0    (-+-------------------)
    2    4   8    39.5    14.8             (-------------+----------)
                                --------+---------+---------+--------
                                     30.0      36.0      42.0
Overall median = 33.5

A 95.0% C.I. for median (1) - median(2): (-17.1,3.1)
```

FIGURE 13.5.1 MINITAB procedure and output for Example 13.5.1.

frequencies that fall above and below the median computed from combined samples. If conditions as to sample size and expected frequencies are met, X^2 may be computed and compared with the critical χ^2 with $k-1$ degrees of freedom.

Computer Analysis The median test calculations may be carried out using MINITAB. To illustrate using the data of Example 13.5.1 we first store the measurements in MINITAB Column 1. In MINITAB Column 2 we store codes that identify the observations as to whether they are for an urban (1) or rural (2) subject. The MINITAB procedure and output are shown in Figure 13.5.1.

EXERCISES

13.5.1 Fifteen patient records from each of two hospitals were reviewed and assigned a score designed to measure level of care. The scores were as follows:

Hospital A:	99, 85, 73, 98, 83, 88, 99, 80, 74, 91, 80, 94, 94, 98, 80
Hospital B:	78, 74, 69, 79, 57, 78, 79, 68, 59, 91, 89, 55, 60, 55, 79

Would you conclude, at the .05 level of significance, that the two population medians are different? Determine the p value.

13.5.2 The following serum albumin values were obtained from 17 normal and 13 hospitalized subjects:

Serum Albumin (g/100 ml)				Serum Albumin (g/100 ml)			
Normal Subjects		Hospitalized Subjects		Normal Subjects		Hospitalized Subjects	
2.4	3.0	1.5	3.1	3.4	4.0	3.8	1.5
3.5	3.2	2.0	1.3	4.5	3.5	3.5	
3.1	3.5	3.4	1.5	5.0	3.6		
4.0	3.8	1.7	1.8	2.9			
4.2	3.9	2.0	2.0				

Would you conclude at the .05 level of significance that the medians of the two populations sampled are different? Determine the p value.

13.6 THE MANN–WHITNEY TEST

The median test discussed in the preceding section does not make full use of all the information present in the two samples when the variable of interest is measured on at least an ordinal scale. Reducing an observation's information content to merely that of whether or not it falls above or below the common median is a waste of information. If, for testing the desired hypothesis, there is available a procedure that makes use of more of the information inherent in the data, that procedure should be used if possible. Such a nonparametric procedure that can often be used instead of the median test is the Mann–Whitney test (5), sometimes called the Mann–Whitney–Wilcoxon test. Since this test is based on the ranks of the observations, it utilizes more information than does the median test.

Assumptions The assumptions underlying the Mann–Whitney test are as follows:

1. The two samples, of size n and m, respectively, available for analysis have been independently and randomly drawn from their respective populations.
2. The measurement scale is at least ordinal.
3. The variable of interest is continuous.
4. If the populations differ at all, they differ only with respect to their medians.

Hypotheses When these assumptions are met we may test the null hypothesis that the two populations have equal medians against either of the three possible alternatives: (1) the populations do not have equal medians (two-sided test), (2) the median of population 1 is larger than the median of population 2 (one-sided test), or (3) the median of population 1 is smaller than the median of population 2 (one-sided test). If the two populations are symmetric, so that within each population the mean and median are the same, the conclusions we reach regarding the two population medians will also apply to the two population means. The following example illustrates the use of the Mann–Whitney test.

Mann–Whitney-Wilcoxon test. Indeed, many computer packages give the test value of both the Mann–Whitney test (U) and the Wilcoxon test (W). These two tests are algebraically equivalent tests, and are related by the following equality when there are no ties in the data:

$$U + W = \frac{m(m + 2n + 1)}{2} \tag{13.6.3}$$

Computer Analysis Many statistics software packages will perform the Mann–Whitney test. With the data of two samples stored in Columns 1 and 2, for example, MINITAB will perform a one-sided or two-sided test. The MINITAB procedure and output for Example 13.6.1 are shown in Figure 13.6.2.

The SPSS output for Example 13.6.1 is shown in Figure 13.6.3. As we see this output provides the Mann–Whitney test, the Wilcoxon test, and large-sample z approximation.

EXERCISES

13.6.1 Cranor and Christensen (A-4) studied diabetics insured by two employers. Group 1 subjects were employed by the City of Asheville, North Carolina, and group 2 subjects were employed by Mission–St. Joseph's Health System. At the start of the study, the researchers performed the Mann–Whitney test to determine if a significant difference in weight existed between the two study groups. The data are displayed in the following table.

Weight (Pounds)					
Group 1			Group 2		
252	215	240	185	195	220
240	190	302	310	210	295
205	270	312	212	190	202
200	159	126	238	172	268
170	204	268	184	190	220
170	215	215	136	140	311
320	254	183	200	280	164
148	164	287	270	264	206
214	288	210	200	270	170
270	138	225	212	210	190
265	240	258	182	192	
203	217	221	225	126	

Source: Data provided courtesy of Carole W. Carnor, Ph.D.

May we conclude, on the basis of these data, that patients in the two groups differ significantly with respect to weight? Let $\alpha = .05$.

13.6.2 One of the purposes of a study by Liu et al. (A-5) was to determine the effects of MRZ 2/579 (a receptor antagonist shown to provide neuroprotective activity in vivo and in vitro) on neurological

Data:

```
C1:  12.22 28.44 28.13 38.69 54.91 3.68 4.05 6.47 21.12 3.33 54.36 27.87 66.81 46.27 30.19
C2:  1 1 1 1 1 2 2 2 2 2 3 3 3 3 3
```

Dialog box:

Stat ➤ Nonparametrics ➤ Kruskal–Wallis
Type *C1* in **Response** and *C2* in **Factor.** Click **OK.**

Session command:

```
MTB > Kruskal-Wallis C1 C2.
```

Output:

Kruskal–Wallis Test: C1 versus C2

```
Kruskal-Wallis Test on C1

C2           N    Median   Ave Rank       Z
1            5    28.440        9.4    0.86
2            5     4.050        3.2   -2.94
3            5    46.270       11.4    2.08
Overall     15                  8.0

H = 9.14  DF = 2  P = 0.010
```

FIGURE 13.8.1 MINITAB procedure and output, Kruskal–Wallis test of eosinophil count data in Table 13.8.1.

EXERCISES

For the following exercises, perform the test at the indicated level of significance and determine the *p* value.

13.8.1 In a study of healthy subjects grouped by age (Younger: 19–50 years, Seniors: 65–75 years, and Longeval: 85–102 years), Herrmann et al. (A-8) measured their vitamin B-12 levels (ng/L). All elderly subjects were living at home and able to carry out normal day-to-day activities. The following table shows vitamin B-12 levels for 50 subjects in the young group, 92 seniors, and 90 subjects in the longeval group.

Young (19–50 Years)		Senior (65–75 Years)				Longeval (85–102 Years)			
230	241	319	371	566	170	148	149	631	198
477	442	190	460	290	542	1941	409	305	321
561	491	461	440	271	282	128	229	393	2772
347	279	163	520	308	194	145	183	282	428
566	334	377	256	440	445	174	193	273	259
260	247	190	335	238	921	495	161	157	111

(*Continued*)

Young (19–50 Years)		Senior (65–75 Years)				Longeval (85–102 Years)			
300	314	375	137	525	1192	460	400	1270	262
230	254	229	452	298	748	548	348	252	161
215	419	193	437	153	187	198	175	262	1113
260	335	294	236	323	350	165	540	381	409
349	455	740	432	205	1365	226	293	162	378
315	297	194	411	248	232	557	196	340	203
257	456	780	268	371	509	166	632	370	221
536	668	245	703	668	357	218	438	483	917
582	240	258	282	197	201	186	368	222	244
293	320	419	290	260	177	346	262	277	
569	562	372	286	198	872	239	190	226	
325	360	413	143	336		240	241	203	
275	357	685	310	421		136	195	369	
172	609	136	352	712		359	220	162	
2000	740	441	262	461		715	164	95	
240	430	423	404	631		252	279	178	
235	645	617	380	1247		414	297	530	
284	395	985	322	1033		372	474	334	
883	302	170	340	285		236	375	521	

Source: Data provided courtesy of W. Herrmann and H. Schorr.

May we conclude, on the basis of these data, that the populations represented by these samples differ with respect to vitamin B-12 levels? Let $\alpha = .01$.

13.8.2 The following are outpatient charges $(-\$100)$ made to patients for a certain surgical procedure by samples of hospitals located in three different areas of the country:

Area		
I	**II**	**III**
\$80.75	\$58.63	\$84.21
78.15	72.70	101.76
85.40	64.20	107.74
71.94	62.50	115.30
82.05	63.24	126.15

Can we conclude at the .05 level of significance that the three areas differ with respect to the charges?

13.8.3 A study of young children by Flexer et al. (A-9) published in the *Hearing Journal* examines the effectiveness of an FM sound field when teaching phonics to children. In the study, children in a classroom with no phonological or phonemic awareness training (control) were compared to a class with phonological and phonemic awareness (PPA) and to a class that utilized phonological and phonemic awareness training and the FM sound field (PPA/FM). A total of 53 students from three separate preschool classrooms participated in this study. Students were given a measure of phonemic awareness in preschool and then at the end of the first semester of kindergarten. The improvement scores are listed in the following table as measured by the Yopp–Singer Test of Phonemic Segmentation.

36.4046	59.15
39.3916	61.7086
39.7583	65.5508
41.8931	69.0863
43.7823	69.9415
47.136	73.2952
48.173	73.477

The median of these averages, 53.1432, is the estimator $\hat{\alpha}_{2,M}$. The estimating equation, then, is $y_i = 53.1432 + .4878x_i$ if we are willing to assume that the distribution of error terms is symmetric about 0. If we are not willing to make the assumption of symmetry, the estimating equation is $y_i = 51.5267 + .4878x_i$. ■

EXERCISES

13.11.1 The following are the heart rates (HR: beats/minute) and oxygen consumption values (VO_2: cal/kg/24 h) for nine infants with chronic congestive heart failure:

HR(X):	163	164	156	151	152	167	165	153	155
VO_2(Y):	53.9	57.4	41.0	40.0	42.0	64.4	59.1	49.9	43.2

Compute $\hat{\beta}_1$, $(\hat{\beta}_0)_{1,M}$, and $(\hat{\beta}_0)_{2,M}$

13.11.2 The following are the body weights (grams) and total surface area (cm^2) of nine laboratory animals:

Body weight (X):	660.2	706.0	924.0	936.0	992.1	888.9	999.4	890.3	841.2
Surface area (Y):	781.7	888.7	1038.1	1040.0	1120.0	1071.5	1134.5	965.3	925.0

Compute the slope estimator and two intercept estimators.

13.12 SUMMARY

This chapter is concerned with nonparametric statistical tests. These tests may be used either when the assumptions underlying the parametric tests are not realized or when the data to be analyzed are measured on a scale too weak for the arithmetic procedures necessary for the parametric tests.

Nine nonparametric tests are described and illustrated. Except for the Kolmogorov–Smirnov goodness-of-fit test, each test provides a nonparametric alternative to a well-known parametric test. There are a number of other nonparametric tests available. The interested reader is referred to the many books devoted to nonparametric methods, including those by Gibbons (14) and Pett (15).