We set the right-hand sides of these equations equal to each other and solve for $n$, to obtain

$$
\begin{equation*}
n=\left[\frac{\left(z_{0}+z_{1}\right) \sigma}{\left(\mu_{0}-\mu_{1}\right)}\right]^{2} \tag{7.10.3}
\end{equation*}
$$

To find $n$ for our illustrative example, we substitute appropriate quantities into Equation 7.10.3. We have $\mu_{0}=65, \mu_{1}=55$, and $\sigma=15$. From Appendix Table D , the value of $z$ that has .01 of the area to its left is -2.33 . The value of $z$ that has .05 of the area to its right is 1.645 . Both $z_{0}$ and $z_{1}$ are taken as positive. We determine whether $C$ lies above or below either $\mu_{0}$ or $\mu_{1}$ when we substitute into Equations 7.10.1 and 7.10.2. Thus, we compute

$$
n=\left[\frac{(2.33+1.645)(15)}{(65-55)}\right]^{2}=35.55
$$

We would need a sample of size 36 to achieve the desired levels of $\alpha$ and $\beta$ when we choose $\mu_{1}=55$ as the alternative value of $\mu$.

We now compute $C$, the critical value for the test, and state an appropriate decision rule. To find $C$, we may substitute known numerical values into either Equation 7.10.1 or Equation 7.10.2. For illustrative purposes, we solve both equations for $C$. First we have

$$
C=65-2.33\left(\frac{15}{\sqrt{36}}\right)=59.175
$$

From Equation 7.10.2, we have

$$
C=55-1.645\left(\frac{15}{\sqrt{36}}\right)=59.1125
$$

The difference between the two results is due to rounding error.
The decision rule, when we use the first value of $C$, is as follows:
Select a sample of size 36 and compute $\bar{x}$, if $\bar{x} \leq 59.175$, reject $H_{0}$. If $\bar{x}>59.175$, do not reject $H_{0}$.

We have limited our discussion of the type II error and the power of a test to the case involving a population mean. The concepts extend to cases involving other parameters.

## EXERCISES

7.10.1 Given $H_{0}: \mu=516, H_{\mathrm{A}}: \mu>516, \quad n=16, \sigma=32, \alpha=.05$. Let $\beta=.10$ and $\mu_{1}=520$, and find $n$ and $C$. State the appropriate decision rule.
7.10.2 Given $H_{0}: \mu \leq 4.500, \quad H_{\mathrm{A}}: \mu>4.500, \quad n=16, \sigma=.020, \alpha=.01$. Let $\beta=.05$ and $\mu_{1}=4.52$, and find $n$ and $C$. State the appropriate decision rule.
7.10.3 Given $H_{0}: \mu \leq 4.25, \quad H_{\mathrm{A}}: \mu>4.25, \quad n=81, \sigma=1.8, \alpha=.01$. Let $\beta=.03$ and $\mu_{1}=5.00$, and find $n$ and $C$. State the appropriate decision rule.

In Exercise 8.2.1 to 8.2.7, go through the ten steps of analysis of variance hypothesis testing to see if you can conclude that there is a difference among population means. Let $\alpha=.05$ for each test. Use Tukey's HSD procedure to test for significant differences among individual pairs of means (if appropriate). Use the same $\alpha$ value for the $F$ test. Construct a dot plot and side-by-side boxplots of the data.
8.2.1. Researchers at Case Western Reserve University (A-2) wanted to develop and implement a transducer, manageable in a clinical setting, for quantifying isometric moments produced at the elbow joint by individuals with tetraplegia (paralysis or paresis of all four limbs). The apparatus, called an elbow moment transducer (EMT), measures the force the elbow can exert when flexing. The output variable is voltage. The machine was tested at four different elbow extension angles, 30, 60, 90 , and 120 degrees, on a mock elbow consisting of two hinged aluminum beams. The data are shown in the following table.

Elbow Angle (Degrees)

| 30 |  | 60 |  | 90 |  |  | 120 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -0.003 | 1.094 | 0.000 | -0.001 | 0.000 | -0.007 | 0.558 | 0.003 |
| 0.050 | 1.061 | 0.053 | 0.010 | 0.006 | 0.012 | 0.529 | 0.062 |
| 0.272 | 1.040 | 0.269 | 0.028 | 0.026 | -0.039 | 0.524 | 0.287 |
| 0.552 | 1.097 | 0.555 | 0.055 | 0.053 | -0.080 | 0.555 | 0.555 |
| 1.116 | 1.080 | 1.103 | 0.105 | 0.108 | -0.118 | 0.539 | 1.118 |
| 2.733 | 1.051 | 2.727 | 0.272 | 0.278 | -0.291 | 0.536 | 2.763 |
| 0.000 | 1.094 | -0.002 | 0.553 | 0.555 | -0.602 | 0.557 | 0.006 |
| 0.056 | 1.075 | 0.052 | 0.840 | 0.834 | -0.884 | 0.544 | 0.050 |
| 0.275 | 1.035 | 0.271 | 1.100 | 1.106 | -1.176 | 0.539 | 0.277 |
| 0.556 | 1.096 | 0.550 | 1.647 | 1.650 | -1.725 | 1.109 | 0.557 |
| 1.100 | 1.100 | 1.097 | 2.728 | 2.729 | 0.003 | 1.085 | 1.113 |
| 2.723 | 1.096 | 2.725 | -0.001 | 0.005 | 0.003 | 1.070 | 2.759 |
| -0.003 | 1.108 | 0.003 | 0.014 | -0.023 | -0.011 | 1.110 | 0.010 |
| 0.055 | 1.099 | 0.052 | 0.027 | -0.037 | -0.060 | 1.069 | 0.060 |
| 0.273 | 1.089 | 0.270 | 0.057 | -0.046 | -0.097 | 1.045 | 0.286 |
| 0.553 | 1.107 | 0.553 | 0.111 | -0.134 | -0.320 | 1.110 | 0.564 |
| 1.100 | 1.094 | 1.100 | 0.276 | -0.297 | -0.593 | 1.066 | 1.104 |
| 2.713 | 1.092 | 2.727 | 0.555 | -0.589 | -0.840 | 1.037 | 2.760 |
| 0.007 | 1.092 | 0.022 | 0.832 | -0.876 | -1.168 | 2.728 | -0.003 |
| -0.066 | 1.104 | -0.075 | 1.099 | -1.157 | -1.760 | 2.694 | -0.060 |
| -0.258 | 1.121 | -0.298 | 1.651 | -1.755 | 0.004 | 2.663 | -0.289 |
| -0.581 | 1.106 | -0.585 | 2.736 | -2.862 | 0.566 | 2.724 | -0.585 |
| -1.162 | 1.135 | -1.168 | 0.564 | 0.000 | 1.116 | 2.693 | -1.180 |
| 0.008 | 1.143 | 0.017 | 0.556 | 0.245 | 2.762 | 2.670 | 0.000 |
| -0.045 | 1.106 | -0.052 | 0.555 | 0.497 | 0.563 | 2.720 | -0.034 |
| -0.274 | 1.135 | -0.258 | 0.567 | 0.001 | 0.551 | 2.688 | -0.295 |
| -0.604 | 1.156 | -0.548 | 0.559 | 0.248 | 0.551 | 2.660 | -0.579 |
| -1.143 | 1.112 | -1.187 | 0.551 | 0.498 | 0.561 | 0.556 | -1.165 |
| -0.004 | 1.104 | 0.019 | 1.107 | 0.001 | 0.555 | 0.560 | -0.019 |

Elbow Angle (Degrees)

| 30 |  | 60 |  | 90 |  |  | 120 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -0.050 | 1.107 | -0.044 | 1.104 | 0.246 | 0.558 | 0.557 | -0.056 |
| -0.290 | 1.107 | -0.292 | 1.102 | 0.491 | 0.551 | 0.551 | -0.270 |
| -0.607 | 1.104 | -0.542 | 1.112 | 0.001 | 0.566 | 0.564 | -0.579 |
| $-1.164$ | 1.117 | $-1.189$ | 1.103 | 0.262 | 0.560 | 0.555 | -1.162 |
| 1.105 | 1.101 |  | 1.104 | 0.527 | 1.107 | 0.551 |  |
|  | 1.103 |  | 1.114 | 0.001 | 1.104 | 0.563 |  |
|  |  |  | 1.095 | 0.260 | 1.109 | 0.559 |  |
|  |  |  | 1.100 | 0.523 | 1.108 | 1.113 |  |
|  |  |  | 2.739 | -0.005 | 1.106 | 1.114 |  |
|  |  |  | 2.721 | 0.261 | 1.102 | 1.101 |  |
|  |  |  | 2.687 | 0.523 | 1.111 | 1.113 |  |
|  |  |  | 2.732 | 2.696 | 1.102 | 1.113 |  |
|  |  |  | 2.702 | 2.664 | 1.107 | 1.097 |  |
|  |  |  | 2.660 | 2.722 | 2.735 | 1.116 |  |
|  |  |  | 2.743 | 2.686 | 2.733 | 1.112 |  |
|  |  |  | 2.687 | 2.661 | 2.659 | 1.098 |  |
|  |  |  | 2.656 | 0.548 | 2.727 | 2.732 |  |
|  |  |  | 2.733 | 2.739 | 0.542 | 2.722 |  |
|  |  |  | 2.731 | 2.742 | 0.556 | 2.734 |  |
|  |  |  |  |  | 2.728 | 2.747 |  |

Source: Data provided courtesy of S. A. Snyder, M.S.
8.2.2. Patients suffering from rheumatic diseases or osteoporosis often suffer critical losses in bone mineral density (BMD). Alendronate is one medication prescribed to build or prevent further loss of BMD. Holcomb and Rothenberg (A-3) looked at 96 women taking alendronate to determine if a difference existed in the mean percent change in BMD among five different primary diagnosis classifications. Group 1 patients were diagnosed with rheumatoid arthritis (RA). Group 2 patients were a mixed collection of patients with diseases including lupus, Wegener's granulomatosis and polyarteritis, and other vasculitic diseases (LUPUS). Group 3 patients had polymyalgia rheumatica or temporal arthritis (PMRTA). Group 4 patients had osteoarthritis (OA) and group 5 patients had osteoporosis $(\mathrm{O})$ with no other rheumatic diseases identified in the medical record. Changes in BMD are shown in the following table.

Diagnosis

|  | RA |  | LUPUS | PMRTA | OA |
| ---: | ---: | ---: | ---: | ---: | ---: |
| 11.091 | 7.412 | 2.961 | -3.669 | 11.146 | O |
| 24.414 | 5.559 | 0.293 | -7.816 | -0.838 | 15.968 |
| 10.025 | 4.761 | 8.394 | 4.563 | 4.082 | 5.349 |
| -3.156 | -3.527 | 2.832 | -0.093 | 6.645 | 1.719 |
| 6.835 | 4.839 | -1.369 | -0.185 | 4.329 | 6.445 |
| 3.321 | 1.850 | 11.288 | 1.302 | 1.234 | 20.243 |
| 1.493 | -3.933 | 3.997 | 5.299 | -2.817 | 3.290 |
|  |  |  |  |  | (Continued) |

Diagnosis

| RA |  | LUPUS | PMRTA | OA | O |
| ---: | ---: | ---: | ---: | ---: | ---: |
| -1.864 | 9.669 | 7.260 | 10.734 | 3.544 | 8.992 |
| 5.386 | 4.659 | 5.546 | 1.399 | 4.160 | 6.120 |
| 3.868 | 1.137 |  | 0.497 | 1.160 | 25.655 |
| 6.209 | 7.521 |  | 0.592 | -0.247 |  |
| -5.640 | 0.073 |  | 3.950 | 5.372 |  |
| 3.514 | -8.684 |  | 0.674 | 6.721 |  |
| -2.308 | -0.372 |  | 9.354 | 9.950 |  |
| 15.981 | 21.311 |  | 2.610 | 10.820 |  |
| -9.646 | 10.831 |  | 5.682 | 7.280 |  |
| 5.188 | 3.351 |  |  | 6.605 |  |
| -1.892 | 9.557 |  |  | 7.507 |  |
| 16.553 |  |  |  | 5.075 |  |
|  |  |  |  | 0.163 |  |
|  |  |  |  | 12.767 |  |
|  |  |  |  | 3.481 |  |
|  |  |  |  | 0.917 |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

Source: Data provided courtesy of John P. Holcomb, Ph.D. and Ralph J. Rothenberg, M.D.
8.2.3. Ilich-Ernst et al. (A-4) investigated dietary intake of calcium among a cross section of 113 healthy women ages 20-88. The researchers formed four age groupings as follows: Group A, 20.0-45.9 years; group B, 46.0-55.9 years; group C, 56.0-65.9 years; and group D, over 66 years. Calcium from food intake was measured in $\mathrm{mg} /$ day. The data below are consistent with summary statistics given in the paper.

| Age Groups (Years) |  |  |  | Age Groups (Years) |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | в | c | D | A | в | c | D |
| 1820 | 191 | 724 | 1652 |  |  | 1020 | 775 |
| 2588 | 1098 | 613 | 1309 |  |  | 805 | 1393 |
| 2670 | 644 | 918 | 1002 |  |  | 631 | 533 |
| 1022 | 136 | 949 | 966 |  |  | 641 | 734 |
| 1555 | 1605 | 877 | 788 |  |  | 760 | 485 |
| 222 | 1247 | 1368 | 472 |  |  |  | 449 |
| 1197 | 1529 | 1692 | 471 |  |  |  | 236 |
| 1249 | 1422 | 697 | 771 |  |  |  | 831 |
| 1520 | 445 | 849 | 869 |  |  |  | 698 |
| 489 | 990 | 1199 | 513 |  |  |  | 167 |
| 2575 | 489 | 429 | 731 |  |  |  | 824 |
| 1426 | 2408 | 798 | 1130 |  |  |  | 448 |
| 1846 | 1064 | 631 | 1034 |  |  |  | 991 |
| 1088 | 629 | 1016 | 1261 |  |  |  | 590 |
| 912 |  | 1025 | 42 |  |  |  | 994 |
| 1383 |  | 948 | 767 |  |  |  | 1781 |

