Math 6397 – Fall 2009 Name:

HOMEWORK #2

(1) Prove that if ψ is a wavelet arising from an MRA then

$$\sum_{k\in\mathbb{Z}}\sum_{j\geq 1}|\hat{\psi}(2^j(\xi+2\pi k))|^2 = 1 \quad a.e.$$

(Hint: use scaling equation in the Fourier domain).

(2) Let ϕ be the scaling function associated with an MRA $\{V_j, j\in\mathbb{Z}\}$ and m_0 the low pass filter. Let

$$S = \{f \in L^2(\mathbb{R}) : \hat{f}(\xi) = e^{i\xi}\, s(2\xi)\,\overline{m_0(\xi+\pi)}\hat{\phi}(\xi),\ s\in L^2(\mathbb{T})\}.$$

Show directly that the space S is perpendicular to V_{-1}.

(Hint: recall that $V_{-1} = \{f \in L^2(\mathbb{R}) : \hat{f}(\xi) = m(2\xi)\, m_0(\xi)\hat{\phi}(\xi),\ m \in L^2(\mathbb{T})\}$)

(3) Let $\phi \in L^1$ be the scaling function associated with an MRA and suppose that ϕ has compact support. Show that

$$\sum_{n\in\mathbb{Z}}\phi(x+n) = 1$$

(Hint: Consider the Fourier coefficients $\int_0^1 e^{-2\pi i k\xi}\sum_n \phi(x+n)\,dx$. Notice that you must justify why this Fourier integral is well defined).