Math 3321 – Spring 2024 Name: SOLUTION

Homework #3

You must justify all steps to get credit for your work

Please submit the HW via CASA or email your completed assignment as a single PDF file to `jshi24@CougarNet.UH.EDU`.

(1)[4Pts] Find the general solution of the following differential equation

$$y'' - 6y' + 9y = 0$$

Characteristic polynomial:

$$p(r) = (r-3)^2$$

Roots: $r_1 = r_2 = 3$
General solution:

$$y = Ae^{3x} + Bxe^{3x}$$

(2)[4Pts] Find the general solution of the following differential equation

$$y'' - 4y' + 5y = 0$$

Characteristic polynomial:

$$p(r) = r^2 - 4r + 5$$

Roots: $r_1, r_2 = 2 \pm i$
General solution:

$$y = (A\cos(x) + B\sin(x))e^{2x}$$

(3)[4Pts] Find the general solution of the following differential equation

$$y'' + y' + \frac{1}{4}y = 0$$

Characteristic polynomial:

$$p(r) = r^2 + r + 1/4$$

Roots: $r_1 = r_2 = -1/2$
General solution:

$$y = Ae^{-x/2} + Bxe^{-x/2}$$

(4)[4Pts] Find the general solution of the homogeneous differential equation and solve the following IVP

$$y'' + 10y = 0, \quad y(0) = 5, y'(0) = 5/\sqrt{10}$$

Characteristic polynomial:

$$p(r) = r^2 + 10$$

Roots: $r_1, r_2 = \pm i\sqrt{10}$
General solution:

$$y = A\cos(\sqrt{10}x) + B\sin(\sqrt{10}x)$$

From the expression of $y(x) = A\cos(\sqrt{10}x) + B\sin(\sqrt{10}x)$*, we compute*

$$y'(x) = -\sqrt{10}A\sin(\sqrt{10}x) + \sqrt{10}B\cos(\sqrt{10}x)$$

By $y(0) = 5$ *and* $y'(0) = 5/\sqrt{10}$*, we have*

$$\begin{cases} A + 0 = 5 \\ 0 + \sqrt{10}B = 5/\sqrt{10} \end{cases} \Rightarrow \begin{cases} A = 5 \\ B = \frac{1}{2} \end{cases}$$

IVP solution

$$y = 5\cos(\sqrt{10}x) + \frac{1}{2}\sin(\sqrt{10}x)$$

(5)[4Pts] Consider the following linear second-order differential equation

$$y'' + \tfrac{1}{x}y' - \tfrac{1}{x^2}y = 0, \qquad x > 0$$

(a) Show that there are 2 solutions of the form x^r*.*
(b) Prove that the 2 solutions are linearly independent.
(c) Find the solution satisfying the initial condition $y(1) = 0$, $y'(1) = 1$

(a) By substitution, we find that Cx^r *is a solution for* $r = 1$ *and* $r = -1$*. Hence* $y_1 = x$, $y_2 = \frac{1}{x}$ *and the general solution is:*

$$y = Ax + Bx^{-1}$$

(b) Calculation of Wronskian gives $W = y_1y_2' - y_2y_1' = -2/x \neq 0$*. This implies that the two functions are linearly independent.*

(c) By (a), the general solution is:

$$y = Ax + Bx^{-1}$$

IVP solution:

$$y(x) = \frac{1}{2}(x - \frac{1}{x})$$