

Homework #6

You must justify all steps to get credit for your work

Please submit the HW via CASA or email your completed assignment as a single PDF file to jshi24@CougarNet.UH.EDU.

(1)[4Pts] Compute the Laplace transform of the following functions

(a)  $f(x) = \frac{1}{2}(x+2)^2 e^x$

(b)  $f(x) = \begin{cases} 1 + \cos x & 0 \leq x \leq 2\pi \\ \cos x & x > 2\pi \end{cases}$

(2)[4Pts] Compute the Inverse Laplace transform of the following functions

(a)  $F(s) = \frac{s^2-2}{(s+1)(s-2)(s-3)}$

(b)  $F(s) = 81e^{-5s} \frac{s+2}{s^2(s+9)}$

(3)[4Pts] Consider the following IVP

$$y''' + 3y'' + 3y' + y = 0$$

with  $y''(0) = 1, y'(0) = 0, y(0) = 0$ . (HINT: The Laplace transform of  $y'''$  is  $s^3Y(s) - s^2y(0) - sy'(0) - y''(0)$ ).

(a) Compute the Laplace transform  $Y(s)$  of the solution

(b) Compute the solution  $y(x)$ .

(4)[4Pts] Consider the following IVP

$$y'' + 3y' + 2y = (1 + 3x)u(x)$$

with  $y(0) = 1, y'(0) = 0$ .

(a) Compute the Laplace transform  $Y(s)$  of the solution

(b) Compute the solution  $y(x)$ .

(5)[4Pts] Consider the following IVP

$$y'' + 2y' + 2y = f(x)$$

with  $y(0) = -1, y'(0) = 2$  and  $f(x) = \begin{cases} 1 & 0 \leq x \leq 1 \\ x-1 & x > 1 \end{cases}$

(a) Compute the Laplace transform  $Y(s)$  of the solution

(b) Compute the solution  $y(x)$ .