Math 3321 – Spring 2024 Name: SOLUTION

Homework #8

You must justify all steps to get credit for your work

Please submit the HW via CASA or email your completed assignment as a single PDF file to `jshi24@CougarNet.UH.EDU`.

(1)[3 Pts] Let A be a 3x4 matrix, B be a 4x4 matrix, and C be a 4x5 matrix.
(a) Give the size of AB, if the operation is well defined.
(b) Give the size of AC, if the operation is well defined.
(b) Give the size of ABC, if the operation is well defined.

All operations are well defined.
(a) AB is a 3x4 matrix; (b) AC is a 3x5 matrix; (a) ABC is a 3x5 matrix.

(2)[4 Pts] Consider the following matrices and find the inverse, if it exists. If it does not exist, explain why.

(a) $\begin{pmatrix} 0 & 1 & -1 \\ 1 & 2 & 0 \\ 0 & 2 & -2 \end{pmatrix}$, *(b)* $\begin{pmatrix} 0 & 1 & -1 \\ 1 & 2 & 0 \\ 1 & 3 & -1 \end{pmatrix}$ *(c)* $\begin{pmatrix} 0 & 1 & -1 \\ 1 & 2 & 0 \end{pmatrix}$ *(d)* $\begin{pmatrix} 0 & 2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -2 \end{pmatrix}$

(a) NOT INVERTIBLE since the first row is linearly dependent with the thord row, then the determinant is 0.

(b) NOT INVERTIBLE since the third row is the sum of the first and second row, hence it is linearly dependent with the the other two rows and the determinant is 0.

(c) NOT INVERTIBLE since the matrix is not square

(d) INVERTIBLE since determinant is non-zero. We compute the inverse:

$$\left(\begin{array}{ccc|ccc} 0 & 2 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -2 & 0 & 0 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1/2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/2 \end{array}\right)$$

This shows that the inverse matrix is $\begin{pmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 0 \\ 0 & 0 & -1/2 \end{pmatrix}$

(3)[3Pts] Determine the values of the parameter k *such that the following matrix is invertible*

(a) $\begin{pmatrix} 2 & k & -1 \\ 0 & 1 & -1 \\ 1 & 2 & 0 \end{pmatrix}$ *(b)* $\begin{pmatrix} 2 & k & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 3 \end{pmatrix}$ *(c)* $\begin{pmatrix} 2 & k & -1 \\ 1 & 1 & -1 \\ 3 & 3 & -3 \end{pmatrix}$

(a) We compute the determinant, finding determinant $= 2(2) + (1-k) = 5 - k$*. Hence the matrix is invertible if* $k \neq 5$*.*

(b) ALWAYS INVERTIBLE since the determinant of the matrix is $(2)(1)(3) = 6 \neq 0$*, independently of* k*.*

(c) NEVER INVERTIBLE since the second row is linearly dependent with the third row, then the determinant is 0, independently of k*.*

(4)[3 Pts] Use Cramer's rule to give the value of y for the solution set to the system of equations

$$\begin{aligned} -x+3y+2z &= -1 \\ -4x+y+2z &= -1 \\ -x+y+z &= 1 \end{aligned}$$

Let A be the matrix of coefficient of the system.
We find that $\det A = 1$ and that $\det A_2 = -9$.
Hence, using Cramer's rule $y = \frac{\det A_2}{\det A} = -9$

(5)[3Pts] Consider the vectors

$$v_1 = (1,-1,-3), v_2 = (1,1,-4), v_3 = (0,2,-1), v_4 = (2,0,-7)$$

(a) Is the set $\{v_1, v_2, v_3, v_4\}$ dependent or independent?
(b) If the set $\{v_1, v_2, v_3\}$ is dependent, what is the maximum number of independent vectors?
(c) Is the set $\{v_1, v_2, v_3\}$ dependent or independent?

(a) The set $\{v_1, v_2, v_3, v_4\}$ is dependent since it consists of 4 vectors in R^3
(b) Row reduce:

$$\begin{pmatrix} 1 & -1 & -3 \\ 1 & 1 & -4 \\ 0 & 2 & -1 \\ 2 & 0 & -7 \end{pmatrix} \to \begin{pmatrix} 1 & -1 & -3 \\ 0 & 2 & -1 \\ 0 & 2 & -1 \\ 0 & 2 & -1 \end{pmatrix} \to \begin{pmatrix} 1 & -1 & -3 \\ 0 & 1 & -1/2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Hence there are at most 2 independent vectors.
(c) The set is dependent since, by part (b), there are only 2 independent vectors in the set.

(6)[4Pts] Find eigenvalues and eigenvectors of $A = \begin{pmatrix} -1 & 2 \\ 3 & -2 \end{pmatrix}$.

Characteristic polynomial:

$$= \lambda^2 + 3\lambda - 4$$

Characteristic equation:

$$\lambda^2 + 3\lambda - 4 = 0$$

Eigenvalues:

$$\lambda_1 = 1, \quad \lambda_2 = -4$$

To find eigenvectors, need to solve $(A - \lambda I)x = 0$.
For $\lambda_1 = 1$, we obtain the system

$$\begin{aligned} -2x_1 + 2x_2 &= 0 \\ 3x_1 - 3x_2 &= 0 \end{aligned}$$

which gives the eigenvector $v_1 = (1,1)$.
For $\lambda_2 = -4$, we obtain the system

$$\begin{aligned} 3x_1 + 2x_2 &= 0 \\ 3x_1 + 2x_2 &= 0 \end{aligned}$$

whixg gives the eigenvector $v_2 = (-2/3, 1)$.