HW 1

Please, write clearly and justify your arguments using the theory covered in class to get credit for your work.

(1) [3Pts] Prove that

$$\sum_{i=1}^{n} i^2 = \frac{1}{6}n(n+1)(2n+1) \quad n \in \mathbb{N}.$$

(2) [3Pts] Prove that, for any $n \in \mathbb{N}$, the number $9^n - 4^n$ is divisible by 5.

(3) [3Pts] Prove that, for any $n \geq 4$ the following inequality holds
$$n^2 \leq 2^n.$$

(4) [1Pts] Let $x, y \in \mathbb{R}$ and $\epsilon > 0$. Prove that is $|x-y| \leq \epsilon$, then $|x| \leq |y|+\epsilon$.