

**HW 2**

Please, write clearly and justify your arguments using the theory covered in class to get credit for your work.

(1) [3Pts] Let  $S, T$  be nonempty subsets of  $\mathbb{R}$  and suppose that  $S \subset T$ . Prove that

$$\inf T \leq \inf S \leq \sup S \leq \sup T$$

(2) [3Pts] Let  $S$  be a nonempty and bounded subset of  $\mathbb{R}$ . Prove that  $m = \sup S$  is unique.

(3) [3Pts] Let  $S = \{1 - \frac{1}{n} : n \in \mathbb{N}\}$ . Prove that  $\sup S = 1$  and find the accumulation points of  $S$  if any. Justify your answer.

(4) [3Pts] Let  $X \subset \mathbb{R}$  be nonempty and  $f, g$  be bounded functions defined on  $X$ . Prove that

$$\sup\{f(x) + g(x) : x \in X\} \leq \sup\{f(x) : x \in X\} + \sup\{g(x) : x \in X\}.$$

Give examples to show that the inequality can be either an equality or a strict inequality.

(5) [3Pts] Let  $S \subset \mathbb{R}$  be nonempty. Show that  $S$  is bounded if and only if there exists a closed bounded interval  $I$  such that  $S \subset I$ .