

HW 2

Please, write clearly and justify your arguments using the theory covered in class to get credit for your work.

(1) [3Pts] Let S, T be nonempty subsets of \mathbb{R} and suppose that $S \subset T$. Prove that

$$\inf T \leq \inf S \leq \sup S \leq \sup T$$

(2) [3Pts] Let S be a nonempty and bounded subset of \mathbb{R} . Prove that $m = \sup S$ is unique.

(3) [3Pts] Let $S = \{1 - \frac{1}{n} : n \in \mathbb{N}\}$. Prove that $\sup S = 1$ and find the accumulation points of S if any. Justify your answer.

(4) [3Pts] Let $X \subset \mathbb{R}$ be nonempty and f, g be bounded functions defined on X . Prove that

$$\sup\{f(x) + g(x) : x \in X\} \leq \sup\{f(x) : x \in X\} + \sup\{g(x) : x \in X\}.$$

Give examples to show that the inequality can be either an equality or a strict inequality.

(5) [3Pts] Let $S \subset \mathbb{R}$ be nonempty. Show that S is bounded if and only if there exists a closed bounded interval I such that $S \subset I$.