Math 3333 – SPRING 2018 Name:

HW 3

Please, write clearly and justify your arguments using the theory covered in class to get credit for your work.

(1) [4 Pts]
(a) Let S_1, S_2 be compact subsets of $\mathbb{R}$. Prove that $S_1 \cup S_2$ is also compact.
(b) Find an infinite collection of compact subsets $\{S_n : n \in \mathbb{N}\}$ such that the union $\cup_n S_n$ is not compact. Explain why the resulting set is not compact.

(2) [3 Pts] Prove that the intersection of any collection of compact subsets is also compact.

(3) [4 Pts] Use the definition of convergence to prove the following:
(a) For any real number k, $\lim_{n\to\infty} k/n = 0$
(b) $\lim_{n\to\infty} \frac{3n+1}{n+2} = 3$.

(4) [3 Pts] Show that the sequence $a_n = cos\frac{n\pi}{3}$ is divergent.

(5) [4 Pts]
(a) Let (s_n) be a sequence such that $\lim_{n\to\infty} s_n = 0$ and (t_n) be a bounded sequence. Prove that the sequence $(s_n\, t_n)$ is convergent.
(b) Show by example that the boundedness of (t_n) is necessary in part (a).