

HW 3

Please, write clearly and justify your arguments using the theory covered in class to get credit for your work.

(1) [4 Pts]

- (a) Let S_1, S_2 be compact subsets of \mathbb{R} . Prove that $S_1 \cup S_2$ is also compact.
- (b) Find an infinite collection of compact subsets $\{S_n : n \in \mathbb{N}\}$ such that the union $\cup_n S_n$ is not compact. Explain why the resulting set is not compact.

(2) [3 Pts] Prove that the intersection of any collection of compact subsets is also compact.

(3) [5 Pts] Mark each statement as True or False. If False, show a counter-example. If True, justify your answer.

- (a) Every finite set is compact.
- (b) The set $\{\frac{1}{n} : n \in \mathbb{N}\}$ is compact.
- (c) If S is unbounded then S has an accumulation point.
- (d) If $S \subset \mathbb{R}$ is compact and x is an accumulation point of S , then $x \in S$.
- (e) If $S \subset \mathbb{R}$ is a compact, then there is at least one point in \mathbb{R} that is an accumulation point of S .

(4) [4 Pts] Use the definition of convergence to prove the following:

- (a) For any real number k , $\lim_{n \rightarrow \infty} k/n = 0$
- (b) $\lim_{n \rightarrow \infty} \frac{3n+1}{n+2} = 3$.

(5) [3 Pts] Show that the sequence $a_n = \cos \frac{n\pi}{3}$ is divergent.