HW 3

Please, write clearly and justify your arguments using the theory covered in class to get credit for your work.

(1) [4 Pts] Prove the following.

(a) An accumulation point of a set S is either an interior point of S or a boundary point of S.

(b) A boundary point of a set S is either an accumulation point of S or an isolated point of S.

(2) [5 Pts] Mark each statement as True or False. If False, show a counter-example. If True, justify your answer.

(a) Every finite set is compact.
(b) The set $\{\frac{1}{n} : n \in \mathbb{N}\}$ is compact.
(c) If S is unbounded then S has an accumulation point.
(d) If $S \subset \mathbb{R}$ is compact and x is an accumulation point of S, then $x \in S$.
(e) If $S \subset \mathbb{R}$ is a compact, then there is at least one point in $\mathbb{R}$ that is an accumulation point of S.

(3) [3 Pts] Prove or give a counterexample: If a set S has a maximum and a minimum, then S is a closed set.

(4) [4 Pts]

(a) Let S_1, S_2 be compact subsets of $\mathbb{R}$. Prove that $S_1 \cup S_2$ is also compact.

(b) Find an infinite collection of compact subsets $\{S_n : n \in \mathbb{N}\}$ such that the union $\cup_n S_n$ is not compact. Explain why the resulting set is not compact.

(5) [3 Pts] Using the definition of compactness, prove that the intersection of any collection of compact subsets is also compact.