

HW 3

Please, write clearly and justify your arguments using the theory covered in class to get credit for your work.

(1) [4 Pts] Prove the following.

(a) An accumulation point of a set S is either an interior point of S or a boundary point of S .

(b) A boundary point of a set S is either an accumulation point of S or an isolated point of S .

(2) [5 Pts] Mark each statement as True or False. If False, show a counterexample. If True, justify your answer.

(a) Every finite set is closed.

(b) The set $\{\frac{1}{n} : n \in \mathbb{N}\}$ has no accumulation points.

(c) If S is unbounded then S has an accumulation point.

(d) If $S \subset \mathbb{R}$ is open and x is an accumulation point of S , then $x \in S$.

(e) If $S \subset \mathbb{R}$ is a closed set, then there is at least one point in \mathbb{R} that is an accumulation point of S .

(3) [3 Pts] Prove or give a counterexample: If a set S has a maximum and a minimum, then S is a closed set.

(4) [4 Pts]

(a) Let S_1, S_2 be closed subsets of \mathbb{R} . Prove that $S_1 \cup S_2$ is also closed.

(b) Find an infinite collection of closed subsets $\{S_n : n \in \mathbb{N}\}$ such that the union $\cup_n S_n$ is not closed. Explain why the resulting set is not closed.