Math 3333 – Spring 2018   Name:

HW 4

Please, write clearly and justify all your statements using the material covered in class to get credit for your work.

(1) Prove that
$$\lim_{n\to\infty} \left(\sqrt{n^2 + 1} - n \right) = 0.$$

(2) Prove or give a counterexamples:
(a) If (s_n) and (t_n) are divergent sequences, then $(s_n + t_n)$ diverges.
(b) If (s_n) and (t_n) are divergent sequences, then $(s_n t_n)$ diverges.
(c) If (s_n) and $(s_n + t_n)$ are convergent sequences, then (t_n) converges.

(3) Prove that if (x_n) is a convergent sequence, $(|x_n|)$ is also convergent. Is the converse true?

(4) Suppose that (x_n) is a convergent sequence and (y_n) is a sequence such that, for any $\epsilon > 0$, there exists an $M > 0$ such that $|x_n - y_m| < \epsilon$ for all $n > M$. Does it follow that (y_n) converge? Prove it or find a counterexample.