Math 3333 Name:

HW 4

Please, write clearly and justify all your statements using the material covered in class to get credit for your work.

(1) [4 Pts]

(a) Let (s_n) be a sequence such that $\lim_{n\to\infty} s_n = 0$ and (t_n) be a bounded sequence. Prove that the sequence $(s_n\, t_n)$ is convergent.

(b) Show by example that the boundedness of (t_n) is necessary in part (a). That is, produce an example to show that the sequence $(s_n\, t_n)$ may diverge if (t_n) is not bounded.

(2) Prove that

$$\lim_{n\to\infty} \left(\sqrt{n^2+1} - n\right) = 0.$$

(3)[4 Pts] Prove or give a counterexamples:

(a) If (s_n) and (t_n) are divergent sequences, then $(s_n + t_n)$ diverges.

(b) If (s_n) and (t_n) are divergent sequences, then $(s_n\, t_n)$ diverges.

(c) If (s_n) and $(s_n + t_n)$ are convergent sequences, then (t_n) converges.

(4)[4 Pts] Prove that if (x_n) is a convergent sequence, $(|x_n|)$ is also convergent. Is the converse true?

(5)[4 Pts] Suppose that (x_n) is a convergent sequence and (y_n) is a sequence such that, for any $\epsilon > 0$, there exists an $M > 0$ such that $|x_n - y_m| < \epsilon$ for all $n > M$. Does it follow that (y_n) converge? Prove it or find a counterexample.