Math 3333 Name:

HW 5

Please, write clearly and justify all your statements using the material covered in class to get credit for your work.

(1) [Prob 3(a)] Prove that the sequence below is monotone and bounded. Next find its limit.

$$s_1 = 1, \quad s_{n+1} = \frac{1}{5}(s_n + 7), \quad n \geq 1.$$

(2) [Prob 5] Let (a_n) and (b_n) be monotone sequences. Prove or give a counterexample.

(a) The sequence (c_n) given by $c_n = a_n + b_n$ is monotone.

(b) The sequence (c_n) given by $c_n = a_n\, b_n$ is monotone.

(3) [Prob 6] Recall: A sequence (s_n) is oscillating if $\liminf s_n < \limsup s_n$. Prove or give a counterexample.

(a) Every oscillating sequence has a convergent subsequence.

(b) Every oscillating sequence diverges.

(c) Every divergent sequence oscillates.

(4) [Prob 9] Let (s_n) be a bounded sequence and suppose that $\liminf s_n = \limsup s_n = s$. Prove that (s_n) is convergent and that $\lim s_n = s$.