Math 3333 Name:

HW 5

Please, write clearly and justify all your statements using the material covered in class to get credit for your work.

(1) Prove that

$$\lim_{n\to\infty} \sqrt{n^2+1} - n = 0$$

(2) Prove that if $\lim_{n\to\infty} s_n = \infty$ and if (t_n) is a bounded sequence, then

$$\lim_{n\to\infty} (s_n + t_n) = \infty$$

(3) Prove that if $\lim_{n\to\infty} s_n = \infty$ and $\lim_{n\to\infty} t_n = L > 0$, then

$$\lim_{n\to\infty} (s_n t_n) = \infty$$

(4) Prove that the sequence below is monotone and bounded. Next find its limit.

$$s_1 = 1, \quad s_{n+1} = \frac{1}{5}(s_n + 7), \quad n \geq 1.$$

(5) Let (a_n) and (b_n) be monotone sequences. Prove or give a counterexample.

(a) The sequence (c_n) given by $c_n = a_n + b_n$ is monotone.

(b) The sequence (c_n) given by $c_n = a_n\, b_n$ is monotone.