Math 3333 Name:

HW 6

Please, write clearly and justify all your statements using the material covered in class to get credit for your work.

(1) Find an example of a sequence of real numbers satisfying each set of properties.
(a) Cauchy, but not monotone.
(b) Monotone, but not Cauchy.
(c) Bounded, but not Cauchy.

(2) Prove or give a counterexample.
(a) Every bounded sequence has a Cauchy subsequence.
(b) Every monotone sequence has a bounded subsequence.
(c) Every convergent sequence can be represented as the sum of two oscillating sequences.

(3) Let (s_n) and (t_n) be bounded sequences.
(a) Prove that $\limsup(s_n + t_n) \leq \limsup s_n + \limsup t_n$.
(b) Find an example to show that equality may not hold in part (a).

(4) Show that each series is divergent.
(a) $\sum(-1)^n$

(b) $\sum \frac{n}{2n+1}$

(c) $\sum \cos \frac{n\pi}{2}$

(5) Find the sum of each series.
(a) $\sum_{n=1}^{\infty} \frac{1}{3^n}$

(b) $\sum_{n=1}^{\infty}(-\frac{3}{4})^n$

(c) $\sum_{n=2}^{\infty} \frac{1}{n(n-1)}$

(6) Let (a_n) be a sequence of nonnegative real numbers. Prove that $\sum a_n$ converges iff the sequence of partial sums is bounded.

(7) Determine whether or not the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}+\sqrt{n}}$ converges. Justify your answer.

(8) Let (x_n) be a sequence of real numbers and let $y_n = x_n - x_{n+1}$ for each $n \in \mathbb{N}$.
(a) Prove that the series $\sum_{n=1}^{\infty} y_n$ converges iff the sequence (x_n) converges.
(b) If $\sum_{n=1}^{\infty} y_n$ converges, what is the sum?